CHAPTER 4: MATHEMATICAL LITERACY

Introduction

Mathematical literacy has two major components; fluency in mathematical procedures and the ability to apply mathematical knowledge in solving problems in a variety of contexts. Towards this the learners need to be equipped with competencies comprising of logical reasoning, well-established processes to understand, model, and solve a problem in different ways, as well as interpret the solution in the context of a problem setting, and communicate the solution to others using symbolic and mathematical representations. The Bhutan Education Blueprint 2014-2024 emphasises on equipping students with knowledge, skills, values, and attitudes to nurture young Bhutanese to become socio-economically productive citizens who are able to respond and confidently cope with global challenges. To support this aspiration, the goal of mathematics education in Bhutan are as follows:

- Help students to achieve the mathematical competence required in a GNH-society that is technologically oriented and information rich.
- Develop in students the: understanding of various mathematical concepts and procedural skills; ability to explore variety of strategies for problem solving; and the ability to think and reason logically to enable them to cope with everyday mathematics, and mathematics in the world of work.
- Provide a foundation for those students who may continue studies in Mathematics or other areas requiring strong mathematical background, and also help foster and develop mathematical talent.
- Help students to develop an understanding of the value of mathematics and its usefulness to them, to nurture confidence in their own mathematical ability, and to encourage a continuing interest in Mathematics.

The NEA will measure the attainment of mathematical literacy at various levels of school years to provide reliable and authentic information on the quality of mathematics education in Bhutan at

critical stages of learning. Further, it will help in framing relevant policies for timely interventions aimed at supporting the development of students' mathematical literacy.

This chapter lays out a comprehensive framework for the assessment of 'Mathematical Literacy' for grades III, VI, and IX.

It comprises of the following sections: Assessing 21st century competencies and values, Defining mathematical literacy, Organisation of the domain, Defining mathematical competencies, Assessing the domain and Measuring learning outcomes.

Assessing 21st Century Competencies and Values

- In this framework, the 21st century competencies defined by international agencies such as UNESCO, OECD, the partnership for 21st Century Education and other agencies have been interpreted in the light of nine student attributes aspired for Bhutanese students in the Bhutan Education Blueprint 2014-2024. Among the nine student attributes, five attributes can be assessed through the cognitive instruments used in testing mathematical literacy in alignment with the learning outcomes for each strand. The five attributes are as follows:knowledge and understanding
- intellectual competence
- communicative competence
- leadership competence
- world readiness

1. Knowledge and Understanding

This attribute can be assessed in terms of mathematical knowledge and understanding. The instrument to assess mathematical literacy will include items focused on mathematical knowledge and understanding. This attribute will be measured through the following indicators:

- recalling of information such as fact, definition, term, or a simple procedure, rules and formulae and recognise patterns.
- formulating one-step, well-defined, and straight and simple algorithmic procedural items.

- performing a well-known algorithm, follow a set procedure (perform a clearly defined series of steps).
- representing mathematical situations in different ways and knowing how different representations connect with each other, how they are similar, and how they are different.
- retrieving information from graphs, tables, texts, or other sources.
- applying mathematical facts, rules, algorithms, and structures when finding solutions

2. Intellectual Competence

This attribute can be assessed in terms of mathematical competencies like formulating, applying and interpreting between the real-world and math world. The instrument to assess mathematical literacy will include items focused on these processes.

This attribute will be measured through the following indicators:

- creating a sequential order of instructions to solve multi-step problems and use procedures.
- identifying the mathematical aspects of a problem situated in a real world context and identifying the significant variables.
- representing a situation mathematically, using appropriate variables, symbols, diagrams, and standard models.
- recognising aspects of a problem that correspond with known problems or mathematical concepts, facts or procedures.
- choosing among an array of the most effective computing tools to portray a mathematical relationship inherent in a contextualised problem.
- devising and implementing strategies for finding mathematical solutions
- using mathematical tools, including technology, to help find exact or approximate solutions
- manipulating numbers, graphical and statistical data and information
- making mathematical diagrams, graphs, simulations, and constructions and extracting mathematical information from them
- using and switching between different representations in the process of finding solutions

- using relationships among numbers and different units of measurement, expressions, quantities, and shapes to mathematise problems and work out solutions

3. Communicative Competence

This attribute can be assessed through items that require students to demonstrate mathematical communication. The aspects of communication include students to understand and interpret the information given in the task, and communicate the solution to others using symbolic and mathematical representations. The instrument to assess mathematical literacy will include items assessing mathematical language. Tasks that require students to explain their interpretations of the mathematical solution in the context of the task will also be classified as tasks that assess communicative competence.

This attribute will be measured through the following indicators:

- understanding and interpreting information
- explaining their interpretations of the mathematical solution in a problem context.
- explaining why a mathematical result or conclusion does, or does not, make sense given the context of a problem.
- using sequential ordering to solve multi-step problems and use procedures.
- presenting and extracting information through mathematical diagrams, graphs, simulations constructions etc.
- using algebraic expressions and equations, and geometric representations
- using mathematical language to express mathematical ideas.

4. Leadership Competence

In the Bhutanese context, leadership attributes include knowledge, i.e., understanding of the discipline and of the real world and the ability to generate creative solutions. These attributes will be addressed in the mathematical literacy instrument by including items assessing the understanding of mathematical concepts and creativity in a variety of contexts.

This attribute will be measured through the following indicators:

- their mathematical knowledge and understanding to solve problem in applied and practical situations.
- their ability to identify the problem, explore options for solutions, and reaching the final outcome their capacity to generalise results for a wider context

5. World-readiness

This attribute can be assessed through items based on analytical, reasoning, critical thinking, creativity and problem-solving skills. Instruments demanding broad mathematical knowledge, skills and cognitive demand appropriate to the stage of development will be used to infer information about this attribute.

This attribute will be measured through the following indicators:

- transfer mathematical knowledge to problems set in a context.
- sieve through problems and solution options through broad mathematical knowledge and skills.

Defining Mathematical Literacy

Mathematics is defined as a logical way of studying numbers, shapes, and spaces with the help of a system of symbols and rules to organise them. Another way to look at it is as the study of structure, order and relation, which developed gradually from the practices of counting, measuring, and describing objects. They provide the requisite mathematical language and tools to investigate and explore the world we live in.

There are two branches of mathematics. One as a discipline that can be studied for its intrinsic pleasure, and the other, to explore, understand and communicate with the world around us. However, both are connected by the same mathematical body of knowledge. In this framework, this knowledge is interpreted in terms of mathematical literacy.

Mathematical literacy is defined as an individual's capacity in ***formulating, applying and interpreting*** *mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict a solution. It assists individuals to recognise the role mathematics plays in the world and to make well-founded judgments and decisions as constructive, engaged and reflective citizens*.

Mathematical literacy ...

The term mathematical literacy is used to emphasise that the focus is on acquiring mathematical knowledge and skills, understanding of mathematical concepts and principals, developing a logical way of thinking, and using the mathematical knowledge and skills to solve problems

… is an individual's capacity to…

In order to solve problems certain basic competencies are required. These competencies include capacity to utilize mathematical concepts, devise strategies, mathematise, represent, reason and argument, communicate, using mathematics symbols and tools.

… in formulating, applying and interpreting mathematics…

The process of completing a task involves mathematical competencies of formulating, applying and interpreting mathematical concepts, procedures and skills.

- **formulating** refers to understanding a given task and converting it into a mathematical structure
- **applying** refers to using mathematical competencies to complete the given task
- **interpreting** refers to skills and procedures in analysing and reflecting upon mathematical facts, solutions or conclusions, in the context of the presented task and determine whether the result(s) or conclusion(s) are reasonable and/or useful.

… in a variety of context….

Mathematics can be learnt and applied in a variety of context- personal, local, global and intra-mathematical. These contexts help the student understand, transfer mathematical knowledge and appreciate the role of mathematics in broader sense.

… includes reasoning mathematically …

Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. It involves logical and systematic thinking to arrive at solutions to problems set in the four types of contexts stated above, through patterns and structures and justify the solution.

… using mathematical concepts, procedures, facts and tools …

Problem solving requires using the most appropriate mathematical concepts, procedures and tools to arrive at solution to problems set in a variety of contexts.

… to describe, explain and predict a solution.

The ultimate use of mathematical knowledge and skills is to understand a problem and to either predict or show a solution to problems set in context.

Organisation of the Domain

The mathematical literacy domain will include three aspects:

- **Content**: Mathematical knowledge and skills.
- **Context**: Situations in which a task can be set.
- **Cognitive Processes:** Metacognitive skills required to complete a task.
- **Mathematical Competencies:** Fundamental qualities that underpin mathematical literacy.

1.1.1 *Content*

This refers to the specific mathematical knowledge and skills required to complete a task. Content areas as provided in national curriculum for mathematics in Bhutan will form the basis for deciding the content aspect of mathematical literacy. Towards the end, the content for mathematical literacy will be closely aligned with Bhutan's mathematics curriculum. Mathematics content is organised in five content strands:

- **Number and Operations**
- **Patterns and Algebra**
- **Measurement**
- **Geometry**
- **Data Management and Probability**

For the purpose of NEA, the first two strands are combined to form the 'Number and Algebra' strand, considering the overlap and interrelation between the content strands that enriches the study of each other. In lower grades, algebra focuses on pre-algebraic concepts which can be combined with number and operations.

In Bhutan's NEA, the mathematics content has been divided into four content strands:

- **Number and Algebra**
- **Geometry**
- **Measurement**

- **Data Management and Probability**
- **Number and Algebra**

To make sense of the world around us in terms of mathematics, **quantification** is required. Numbers are fundamental to **quantification**, and different types of numbers add **precision in different ways ─** whole numbers can serve as counters or estimators, fractions and decimals add to the accuracy of measurement, positive and negative numbers act as directional indicators, and percent and ratio aid comparisons. Additionally, numbers can be used to put things in **order** and as **identifiers** (e.g., telephone numbers or zip codes).

Algebra enables a person to **recognise**, **interpret**, and **create patterns** to understand the world better. The pre-algebraic concepts, for example, identifying missing numbers or relationships in simple numeric and geometric patterns, are developed in the primary grades. As students' progress from primary grades to higher grades they build on their understanding of the number system to describe relationships and formulate generalisations. They recognise equivalence and solve equations and inequalities. They apply concept of numbers and the four fundamental operations (+, –, x, ÷) with an increasing degree of complexity and levels of learning and algebra skills, to conduct investigations, solve problems and communicate their reasoning.

The 'Number and Algebra' strand can be further divided into subsections – whole numbers, fraction and decimal, integers, rational numbers, and patterns and algebra. The proportional coverage of number and algebra is higher in grade III and grade VI than for grade IX.

Based on the curriculum coverage of number and algebra at the end of key stages 1, 2 and 4 the approximate proportion is given in the table 4.1.

Table 4.1: Number and Algebra coverage in the Mathematical Literacy Assessment

Grade	Approximate percentage coverage
III	60% 55-65%
VI	50% 45-55%
IX	40% 35-45%

Geometry

Geometry, the study of shape, space and spatial relationships, is an important and essential branch of mathematics. Shape is a category describing **real images and entities that can be visualised in both two and three dimensions**, e.g., man-made things like buildings, vehicles, art and craft or natural things like snowflakes, shadows and plants. **Direction** and **location** are fundamental qualities which are called upon when reading, interpreting, or sketching maps and diagrams. Geometry is a formal study of shapes and their characteristics and relationships. Through the course of the study, students develop an increasingly sophisticated understanding of **size**, **shape**, **position** and **movement** in two and three dimensions. Understanding of basic geometrical concepts deepens with the analytical study of parallelism, perpendicularity and angle relationships. As learners progress, they start applying their knowledge and understanding to establish relationships between various attributes in shapes and apply them in systematically solving problems. The curriculum coverage of geometry content in primary grades is less in comparison to the secondary grades; taking this into account the coverage of geometry in NEA aligns with the mathematics curriculum in Bhutan as given in the table 4.2.

Table 4.2: Geometry coverage in the Mathematical Literacy Assessment

Grade	Approximate percentage coverage
III	15% 10-20%
VI	20% 15-25%
IX	20% 15-25%

Measurement

Measurement is a tool to **quantify things** around us. This content area requires an understanding of the **units of measure** and the **systems of measurement**, both non-formal and formal standardised ones, such as the Metric and the Imperial systems. A basic task in this fundamental aspect could be – *measure a given quantity* whereas a complex task might involve *describing change in the capacity of an object when one dimension is changed*.

In the mathematics curriculum, students begin with learning about **measurable attributes** of objects and proceed from the non-standard to standard units of measurement. Gradually, the concepts of length, mass, temperature, time, area and volume are learned. Students also build an

understanding of connections between the different units of measures, and calculate derived measures such as area, speed and density.

Table 4.3 gives the approximate proportion of test items in measurement. It aligns with the curriculum coverage of measurement at the end of key stages 1, 2 and 4.

Table 4.3: Measurement coverage in the Mathematical Literacy Assessment

Grade	Approximate percentage coverage
III	15% 10-20%
VI	15% 10-20%
IX	20% 15-25%

Data Management and Probability

In data management statistics, students **collect**, **recognise**, and **organise** data and then draw **inferences**. They learn to **represent**, **summarise** and **interpret** data and undertake purposeful investigations involving the collection and interpretation of data. In probability, they assess likelihood and assign probabilities using **experimental** and **theoretical** approaches. Gradually, the students develop increasingly sophisticated ability to critically evaluate data and probability concepts, and make reasoned judgements and decisions. Since the two components – data management and probability ─ are developed in parallel, they are combined under a single strand of Data Management and Probability for the purpose of Bhutan's NEAF.

Coverage of Data Management and Probability in primary grades is less than the secondary grades.

The approximate proportion of test items in data management and probability is given in the table 4.4. This is based on the curriculum coverage of measurement at the end of key stages 1, 2 and 4.

Table 4.4: Data management and Probability coverage in the Mathematical Literacy Assessment

Grade	Approximate percentage coverage
III	10% 5-15%

VI	15% 10-20%
IX	20% 15-25%

The detailed categorisation ensures adequate coverage of mathematical learning outcomes in the Mathematics Curriculum Framework for Bhutan. It is also designed to provide tasks of appropriate challenge for students across a wide range of proficiency levels.

The distribution for assessment tasks of different content areas is proportional to the content distribution in the curriculum. However, in real life it has to be understood that problems do not necessarily fall neatly into one content category. How a student as a problem solver, chooses knowledge, skills and procedures, appropriate to the problem, combining aspects of different content areas would reflect the general mathematical literacy of the student.

1.1.2 *Context*

Contexts in mathematics are the situations in which a problem to be solved has arisen (MTEG, 2016). It stimulates the task in which mathematical thinking can be assessed. The purpose of defining the contexts, is to ensure that the NEA contains a variety of stimulus material to assess mathematical literacy. It is proposed to assess mathematical literacy through four contexts – personal, local, global and intra-mathematical. In real life these contexts are not totally isolated.

Personal context, the stimulus material for this context have an inward focus, i.e., context is set around individual experiences itself and all interactions affect only the individual. Examples of personal context could be money transactions in buying and selling of commodities by an individual, personal travel, etc.

Local contexts pertain to tasks that require engagement with other individuals or with elements of the surrounding environment. These contexts can be a setting in home, school, community or the nation at large. .

Global contexts require students to go beyond their actual experiences and understand effects on larger communities or communities as a whole. Such contexts require students to go beyond the physical world and develop a degree of abstractness to respond to the tasks. Examples of global contexts could be the height or structure of famous buildings, population, national or regional economic problems etc.

Intra-mathematical contexts, some tasks in mathematics do not have any context and are set in the mathematical domain only. In school mathematics, procedural fluency is considered one of the important skills, thus some contexts in NEA will be purely mathematical.

1.1.3 *Cognitive Processes*

The definition of mathematical literacy refers to an individual's capacity in formulating situations mathematically, applying mathematical concepts, facts, procedures and skills, and interpreting, reasoning and evaluating mathematical outcomes or results. For the purpose of NEA items will be set with focus on either of these three mathematical processes:

- **Formulating** refers to the process of providing a mathematical structure to a real life problem.**,** It indicates ability in recognising and identifying opportunities to use mathematics in a given situation and then providing the necessary mathematical structure needed to translate the given task into the mathematical world.

- **Applying** refers to the process of using mathematical concepts, facts, and procedures to perform computations/manipulations and to arrive at a mathematical solution for a given task.

- **Interpreting** refers to the process of reasoning and reflecting upon mathematical solutions or conclusions, understanding them in the context of a problem and determining whether the result(s) or conclusion(s) are reasonable and/or useful.

Students' capacity at applying mathematics to problems set in various contexts is dependent on competency inherent in all three of the above processes.

Formulating situations mathematically

To succeed in mathematics, a number of cognitive processes need to work together. Basics among them are the student's ability to recall rules and formulae and recognise patterns; use language to understand vocabulary, instructions and explain their thinking; and use sequential ordering to solve multi-step problems and use procedures. Other than this, the students need to learn to use spatial ordering to recognise symbols and deal with geometric forms.

In the mathematical literacy definition, formulate refers to the process of converting a given task into the mathematical language. The task is transferred from a real-world setting to the domain of mathematics and converted into mathematical structures, representations and specificity, bearing in mind the considerations and assumptions given in the task.

The tasks which comes under formulating situations mathematically can be:

:

- Recognising and recalling definitions, number properties, units of measurement, geometric properties, and notations in a given task.
- Selecting an appropriate model from a list.
- Classifying and ordering numbers, expressions, quantities and shapes by common properties.
- Identifying the mathematical aspects of a problem situated in a real-world context and identifying the significant variables.
- Representing a situation mathematically, using appropriate variables, symbols, diagrams, and standard models.
- Retrieving information from graphs, tables, texts, or other sources.
- Representing a problem in different ways, including organising it according to mathematical concepts.
- Recognising aspects of a problem that correspond with known problems or mathematical concepts, facts or procedures.
- Choosing among an array of the most effective computing tools to portray a mathematical relationship inherent in a contextualised problem.
- Creating an ordered series of (step-by-step) instructions for solving problem.

Applying mathematical concepts, facts, and procedures

Applying refers to students' ability to use knowledge and conceptual understanding to solve a problem. Successful problem solving involves the process of coordinating previous experiences, knowledge, and intuition in an effort to determine an outcome of a situation for which a procedure for determining the outcome is not known (Lester, 1987). This skill focusses on students' ability to use mathematics as a tool in familiar situations and routine problems.

In the mathematical literacy definition, ‘apply’ refers to the competency of using mathematical concepts, facts, procedures, and reasoning to solve mathematically-formulated problems in order to obtain mathematical conclusions. In the process of applying mathematical concepts, facts, procedures and reasoning to solve problems ─ individuals perform the mathematical procedures required for finding mathematical solution (e.g. performing arithmetic computations, solving equations, performing symbolic manipulations, extracting mathematical information from tables and graphs, representing and manipulating shapes in space, and simple analysis of data). The tasks which involve this process of applying can be:

- performing a simple calculation
- drawing a simple conclusion
- selecting an appropriate strategy from a list
- devising and implementing strategies for finding mathematical solutions
- using mathematical tools, including technology, to help find exact or approximate solutions
- applying mathematical facts, rules, algorithms, and structures when finding solutions
- manipulating numbers, graphical and statistical data and information
- algebraic expressions and equations, and geometric representations
- making mathematical diagrams, graphs, simulations, and constructions and extracting mathematical information from them
- using and switching between different representations in the process of finding solutions

Interpreting, reasoning and evaluating mathematical outcomes

Reasoning is logically rooted thought processes that explore and link problem elements to make inferences from them; or to check a given justification; or to provide a justification (Turner, 2010). Reasoning mathematically involves logical and systematic thinking. It includes intuitive and inductive reasoning based on the patterns and regularities that can be used to arrive at the solutions

to problems set in novel or unfamiliar situations. Such problems may be purely mathematical or may have real life settings.

The word interpreting used in the mathematical literacy definition focuses on the ability of individuals to reflect upon mathematical solutions, results or conclusions and interpret them in the context of the real-life problem that initiated the process. Interpret in mathematical literacy involve reasoning with the mathematical solution in the context of the problem and also evaluating the reasonableness of the solution or processes in the context of the problem. Interpreting, reasoning and evaluating mathematical outcomes encompasses both the 'reasoning' and 'evaluating' elements of the mathematical modelling cycle. Individuals engaged in this process may be called upon to construct and communicate explanations and arguments in the context of the problem, reflecting on both the modelling process and its results. Specifically, this process of interpreting, reasoning, and evaluating mathematical outcomes includes activities such as:

- Determining, describing, or using relationships among numbers, expressions, quantities, and shapes.
- Evaluating a mathematical outcome in terms of the context.
- Interpreting a mathematical result back into the real-world context.
- Evaluating the reasonableness of a mathematical solution in the context of a given task.
- Understanding how the real world impacts the outcomes and calculations of a mathematical procedure or model in order to make contextual judgments about how the results should be adjusted or applied.
- Explaining why a mathematical result or conclusion does, or does not, make sense given the context of a problem.
- Understanding the extent and limits of mathematical concepts and mathematical solutions.
- Critiquing and identifying the limits of the model used to solve a problem.
- Using mathematical thinking and computational thinking to make predictions, to provide evidence for arguments, to test and compare proposed solutions.
- Reflecting on mathematical arguments and explaining and justifying mathematical results.
- Creating new ideas.

Table 4.5: Proportion of mathematical processes in the Mathematical Literacy Assessment

Mathematical Competencies	Approximate percentage coverage
Formulating	25 - 30%
Applying	45- 50%
Interpreting	25 - 30%

1.1.4 *Mathematical Competencies*

Mathematical competencies are a set of characteristics or qualities that underpins each of the three processes of mathematical literacy in practice, possessed to a greater or lesser extent by a student. Development of mathematical literacy will help students to handle real-world challenges by making productive use of their mathematical knowledge in applied and practical situations. The school mathematics education identifies communication, mathematising, representation, reasoning, devising strategies, and using symbolic, formal and technical language and operations, and mathematical tools as the required mathematical competencies.

Communication comprises of two aspects. Incoming communication refers to reading, decoding, interpreting statements and mathematical information. Outgoing communication refers to explaining, presenting and arguing mathematical results.

Mathematising refers to transforming a real world problem into a mathematical problem. Interpret mathematical objects or information in relation to the situation represented.

Representation is devising or using depictions of mathematical objects or relationships, equations, formulae, graphs, tables, diagrams, textual descriptions.

Reasoning and argument entails developing logically rooted thought processes that explore and link problem elements to make inferences from them; or to check a given justification; or to provide a justification.

Strategic competence refers selecting or devising, and implementing, an appropriate mathematical strategy to solve problems arising from a task or context.

Using symbolic, formal and technical language and operations, and mathematical tools refers to understanding, manipulating, and making use of symbolic expressions, using constructs based on definitions, rules and conventions, formal systems, and relevant mathematical tools including software.

Assessing the Domain

The mathematical literacy domain is comprised of content, context and cognitive processes underpinned by mathematical competencies. The achievement of mathematical literacy will be measured through the analysis of the three processes of formulating, applying and interpreting and related mathematical competencies.

Table 5.2 below shows the relation between the relationship between mathematical processes and the mathematical competencies that underpin these processes.

Table 5.2: Relating competencies to processes

Processes / Competencies	**Formulating situations mathematically**	**Applying mathematical concepts, facts, procedures and skills**	**Interpreting, evaluating and reasoning mathematical outcomes**
Communicating (reading, decoding, interpreting statements and mathematical information, explaining, presenting and arguing)	Read, decode, and make sense of statements, questions, tasks, objects or images, in order to form a mental model of the situation	Explain a solution, Show the work involved in reaching a solution and/or summarise and present intermediate mathematical results	Construct and communicate explanations and arguments in the context of the problem
Mathematising (Transform a real world problem into a mathematical problem.	Identify the underlying mathematical variables and structures in the real world problem	Use an understanding of the context to guide or expedite the mathematical solving process,	Understand the extent and limits of a mathematical solution that are a consequence of the mathematical model employed.

Interpret mathematical objects or information in relation to the situation represented**)**		e.g. working to a context appropriate level of accuracy	
Representation **(**Devising or using depictions of mathematical objects or relationships: equations, formulae, graphs, tables, diagrams, textual descriptions**)**	Create a mathematical representation of real-world information	Make sense of, relate and use a variety of representations when interacting with a problem	Interpret mathematical outcomes in a variety of formats in relation to a situation or use; compare or evaluate two or more representations in relation to a situation
Reasoning and **Argument** **(**Logically rooted thought processes that explore and link problem elements to make inferences from them; or to check a given justification; or to provide a justification**)**	Explain, defend or provide a justification for the identified or devised representation of a real world situation	Explain, defend or provide a justification for the processes and procedures used to determine a mathematical result or solution. Connect pieces of information to arrive at a mathematical solution, make generalisations or create a multi-step argument	Reflect on mathematical solutions and create explanations and arguments that support, refute or qualify a mathematical solution to a contextualised problem
Devising strategies for Solving problems **(**Selecting or devising, and implementing, a mathematical strategy to solve problems arising from the task or context.**)**	Select or devise a plan or strategy to mathematically reframe contextualised problems	Activate effective and sustained control mechanisms across a multi-step procedure leading to a mathematical solution, conclusion or generalisation	Devise and implement a strategy in order to interpret, evaluate and validate a mathematical solution to a contextualised problem
Using symbolic, formal **and technical language** **and operations, and mathematical** **tools** **(**Understanding, manipulating, and making use of symbolic expressions; using constructs based on definitions, rules and conventions, formal	Use appropriate variables, symbols, diagrams and standard models in order to represent a real-world problem using. Use mathematical tools in order to recognise mathematical structures or to portray mathematical relationships	Understand and utilise formal constructs based on definitions, rules and formal systems as well as employing algorithms. Know about and be able to make appropriate use of various tools that may assist in implementing processes and procedures for determining mathematical solutions	Understand the relationship between the context of the problem and representation of the mathematical solution. Use this understanding to help interpret the solution in context and gauge, the feasibility and possible limitations of the solution. Use mathematical tools to ascertain the

systems and using mathematical tools)	symbolic/formal language		reasonableness of a mathematical solution and any limits and constraints on that solution, given the context of the problem

1.1.5 *Item formats*

An assessment instrument for mathematical literacy constitutes an effective stimuli set in a relevant context that demands various levels of cognitive rigour and competencies. It may be either in Multiple Choice Question (MCQ) or Constructed Response Test (CRT) format. For its ease of administration and objective grading, multiple-choice testing is the prevalent form of assessment in science and humanities education (Ali, Carr, & Ruit, 2016). Considering the best international practices, it is proposed that a majority of questions in the test are MCQs. The intended proportion will be the same across grade levels, as provided below.

Table 4.6: Proportion of item types in the Mathematical Literacy Assessment

Type	Grade 3	Grade 6	Grade 9
MCQ	80 – 85%	70 – 80%	60- 70%
CRT	15 – 20%	20- 30%	30- 40%

1.1.6 *Item difficulty*

Research shows that learning is most likely to happen when students are given challenging tasks just beyond their comfort zone. At any given point during their learning, every student is capable of further progress if they can be engaged, motivated and provided with relevant learning opportunities (Masters, 2013). In a particular grade level, all students will not be at the same level of learning. They will be in a range of levels, with the possibility of bright students being several years ahead of the low performing students.

Therefore, to assess the students at all levels of learning, the NEA will include items across a difficulty range. The proportion of these levels of difficulty ─ easy, medium, and difficult ─ is flexible in the first cycle, and will be subsequently fixed during the future cycles. For the first cycle, approximately 20 – 25 percent of the test items will be easy, 50 – 60 percent of items will be at the medium level, and the remaining 20 – 25 percent will be difficult.

Measurable Learning Outcomes

In a practical situation, it is not possible to measure all the LOs prescribed in the curriculum in a single assessment, hence a review of the LOs was undertaken and only the LOs which can be measured in a large-scale assessment have been chosen. In order to ensure that the assessment provides an opportunity for students at different levels of learning to demonstrate their ability, for each grade assessed, appropriate LOs from two grades below and two grades above the targeted grade have been reviewed and included, e.g., LOs for grade VI assessment include LOs from grades 4 to 8.

The table 4.7 shows the list of LOs selected from the Mathematics curriculum framework for the mathematical literacy assessment.

Grade 3
Number and Algebra
Whole Numbers
C3N-1 Represent whole numbers with manipulatives, in pictorial form, words, diagrams, number lines or symbols from 100 to 1,000,000
C3N-2 Demonstrate knowledge of place value of 2-digit to 6-digit numbers
C3N-3 Order and compare numbers using symbols 100 to 1,000,000
C3N-4 Add up to 5-digit numbers with and without regrouping, including computation in simple contextual problems
C3N-5 Subtract up to 5-digit numbers with and without regrouping, including computation in simple contextual problems
C3N-6 Demonstrate knowledge of different addition strategies for 1 to 5-digit numbers: double, half, make ten, front-end, (adding a number from left to right) counting on, subtract 10 and compensate, balancing, using the nearest multiple of ten then compensating, partner number in simple addition Subtraction problems
C3N-7 Demonstrate knowledge of different subtraction strategies for 1 to 5-digit numbers: double, half, make ten, front-end, (adding a number from left to right) counting on, subtract 10 and compensate, balancing, using the nearest multiple of ten then compensating, partner number in simple addition Subtraction problems
C3N-8 Demonstrate the understanding of multiplication as equal grouping and repeated addition, skip counting, double facts, multiplication as array.

C3N-9 Multiply up to 4-digit by 1-digit, including computation in simple contextual problems.
C3N-10 Demonstrate knowledge of division as equal sharing and repeated subtraction and understanding of relation between multiplication and division facts
C3N-11 Demonstrate knowledge of properties of numbers (odd or even) or operations (commutative and associative) to solve problems in simple context
Fractions and Decimals
C3N-12 Identify fractions as parts of a whole or part of a set; represent fractions using words, numbers, or models, including those set in problem situations (Fractions may have denominators of 2, 3, 4, 5, 6, 8, or 10)
C3N-13 Demonstrate knowledge of decimal place value (up to tenth) including representing decimals using words, numbers, or models; compare, order, and round decimals.
Patterns and Algebra
C3N-14 Demonstrate knowledge of different types of patterns (repeating, growing) based on size, shape, colour, attributes etc.
C3N-15 Use understanding of patterns in context (missing number or operation in a number sentence, etc.)
C3N-16 Identify and use relationships in a well-defined numerical and geometrical pattern.
Measurement
C3M-01 Demonstrate the understanding of length, capacity, mass in standard and non-standard units and time using analogue clock (up to 5 minutes) and digital clock.
C3M-02 Solve problems involving length (millimetres, centimetres, meters, and kilometres), mass (gram and kilogram), volume (millilitre, litre) and calendar (days, week, month and seasons): identify appropriate types and sizes of units and read scales
C3M-03 Demonstrate the understanding of relationships among different units of time such as minutes, hours, days, weeks, months, seasons and years
C3M-04 Identify and compare angles based on right angle
Geometry
C3G-01 Identify quarter, half and full turns
C3G-02 Identify, different types of lines (parallel lines, perpendicular lines, etc.), reflection, symmetry and congruence in real world
C3G-03 Identify common 2-D (triangles and quadrilaterals) and 3-D shapes (cube, cuboid, cylinder, cone and sphere) and their attributes
C3G-05 Classify 2-D and 3-D shapes by their attributes

C3G-06 Identify nets of prisms and pyramids (upto hexagonal)
Data Management and Probability
Reading, Interpreting, and Representing Data
C3D-01 Read, represent and interpret data from tally charts, tables, pictographs, and bar graphs
C3D-02 Organize and represent data in tables, pictographs, and bar graphs to help answer questions (simple scales can be included)
C3D-03 Identify and describe probability in terms of impossible, unlikely, equally likely, likely and certain and by using fractions

Grade 6
Number and Algebra
Whole Numbers and Integers
C6N-01 Represent whole numbers with manipulatives, in pictorial form, words, diagrams, number lines or symbols up to 9 places
C6N-02 Demonstrate knowledge of place value of whole numbers up to 9 places
C6N-03 Order and compare whole numbers up to 9 places
C6N-04 Solve problems using addition, subtraction, multiplication and division of whole numbers
C6N-05 Find and use common factors of whole number up to 2-digit
C6N-06 Demonstrate the knowledge of positive and negative integers including on the number line and models
Fractions and Decimals
C6N-07 Demonstrate knowledge of fraction and mixed numbers using diagrams, number lines and models (grid and rectangular)
C6N-08 Order and compare fractions using different strategies (number line, reference number, etc.)
C6N-09 Add and subtract simple fractions (like and unlike fractions, proper and mixed fractions) using different strategies

C6N-10 Demonstrate knowledge of decimals using diagrams, number line, models; order and compare decimal numbers
C6N-11 Add, subtract, multiply and divide decimals with whole numbers using different strategies
C6N-12 Compute with decimals including those set in problem situations
C6N-13 Estimate the product and quotient of decimal multiplication and division
Ratio and Percentage
C6N-14 Demonstrate knowledge of ratio as part to part and part to whole
C6N-15 Use the concept of equivalent ratios and apply it to solve problems using models and symbols
C6N-16 Demonstrate knowledge of percentage as a way to benchmark and estimation for familiar fractions
C6N-17 Demonstrate the understanding of rate by relating them to ratio
Patterns and Algebra
C6N-18 Demonstrate the understanding of function as input/output values
C6N19 Identify and generate patterns with whole numbers (including square and triangular numbers) and decimals
C6N-20 Demonstrate understanding of multiplication and division computation patterns and multiplicative relationship between equivalent fractions
C6N-21 Demonstrate understanding of relation betweendimensions and area/perimeter/volume
C6N-23 Write expressions and equations to represent problem situations
Measurement
C6M-01 Measure and estimate length in SI units (mm, cm, m and km), mass in SI unit (g, kg, tonne)
C6N-02 Demonstrate understanding of relation between different SI units
C6M-03 Demonstrate understanding of angle as measure of turn; identify and draw types of angles, and triangles based on angles
C6M-04 Solve problems involving time including time intervals and elapsed time
C6M-05 Solve problems involving area and perimeter of polygons including triangles, squares, rectangles and parallelograms, use area unit as square centimetre
C6M-06 Estimate and find the volume of prisms
Geometry

C6G-01 Solve problems using angle bisectors, parallel and perpendicular lines
C6G-02 Solve problems using geometric properties of triangles.
C6G-03 Recognize and draw images of geometric transformations (flips, translations, reflections, rotations and tessellations) in the plane
C6G-04 Identify rotational symmetry and use its properties in simple geometric shapes
C6G-05 Identify and draw isometric and orthographic images of geometric shapes
C6G-06 Relate 3-D shapes (prisms and pyramids) with their 2-D representations
C6G-07 Identify planes of symmetry in 3-D shapes

Data Management and Probability
Reading, Interpreting, and Representing Data
C6D-01 Read, interpret, and represent real world data using single and double bar graphs, line graphs, and stem and leaf plots
C6D-02 Calculate and use mean, median and mode of given data
C6D-03 Construct and interpret coordinate graphs
Probability
C6D-04 Identify outcomes as more/less likely
C6D-05 Determine theoretical and experimental probability of simple events

Grade 9
Real Numbers
C9N-01 Demonstrate knowledge of divisibility rules, LCM, and GCF

C9N-02 Solve problems involving ratios, rates, proportion, and percentages including real world problems
C9N-03 Demonstrate knowledge of integer, rational, and irrational numbers including representation, comparing and ordering them
C9N-04 Demonstrate knowledge of exponents including negative exponents and scientific notation
C9N-05 Solve problems involving roots including square roots estimation
C9N-06 Write expressions, equations, or inequalities to represent problem situations and solutions
C9N-07 Represent and solve problems using matrices including networking problems
C9N-08 Demonstrate knowledge of order of operations involving rational numbers
C9N-09 Compute and solve problems with integers, fractions, and decimals (including rational numbers and decimals)
C9N-10 Demonstrate understanding of properties of operations (commutative, associative, and distributive)
C9N-11 Compute with irrational numbers
C9N-12 Solve problems involving simple interest, compound interest, and taxes
C9N-13 Solve problems, analyse situations and make decision involving financing
C9N-14 Identify like and unlike terms
C9N-15 Simplify algebraic expressions including use of commutative, associative and distributive properties
C9N-16 Evaluate polynomial expressions for given values of the variables
C9N-17 Add, subtract, multiply, and divide polynomials
C9N-18 Solve linear and simple radical, exponential, and absolute value equations, linear inequalities, and simultaneous linear equations in two variables, including those that model real life situations, using a number of strategies including graphically.
C9N-19 Interpret, relate and generate representations of linear and non-linear functions in tables, graphs, or words; identify properties of linear functions including slope and intercepts

C9N-20 Solve two linear equations graphically
C9N-21 Solve quadratic equations using factors and graphically
C9N-22 Analyse and describe transformations and apply them to absolute value functions including linear and quadratic functions
C9N-23 Demonstrate understanding of independent and dependent variables, and domain and range
C9N-24 Apply and predict patterns including scatter plots in real world relationships
Measurement
C9M-01 Solve measurement problems involving unit conversion using proportion
C9M-02 Solve problems with diameter, radii, circumference and area of circle
C9M-03 Find area of composite shapes
C9M-04 Demonstrate knowledge of Pythagorean relationship and use it to solve problems
C9M-05 Solve problems involving area and perimeter of quadrilaterals
C9M-06 Calculate volume and surface area of right prism, cylinders, pyramids, cones, spheres and composite 3-D shapes
C9M-07 Demonstrate knowledge of properties of similar triangles and use the knowledge to solve problems
C9M-08 Demonstrate understanding of trigonometric ratios and identities and use the understanding to solve problems
C9M-09 Solve bearing and vector problems using the Pythagorean theorem and/or trigonometric ratios
C9M-10 Recognize that a network with more than two odd vertices is not traversable
Geometry
C9G-01 Use the relationships between angles on lines and in geometric figures to solve problems
C9G-02 Demonstrate the knowledge of altitudes, medians, angle bisectors and perpendicular bisectors

C9G-03 Interpret and analyse properties of geometric transformations (translations, reflections, and rotations) in the plane; identify congruent and similarity criteria in triangles and solve related problems
C9G-04 Represent, analyse and apply concept of dilatations on geometric figures
C9G-05 Use orthographic mat, and isometric drawings to represent more than one 3-D shape
C9G-06 Demonstrate knowledge of minimum sufficient conditions for a unique triangle
C9G-07 Demonstrate understanding of inductive and deductive reasoning
C9G-08 Analyse the relation between number of lines symmetry and rotation to sides of regular polygon
C9G-09 Identify algebraic equation related to transformation and use them to draw graphs
Data Management and Probability
Reading, Interpreting, and Representing Data
C9D-01 Identify appropriate procedures for collecting data, examine biasness in data; organize and represent data including circle graphs, histograms, box and whisker plots, scatter plots to help answer questions and analyse results
C9D-02 Demonstrate an understanding of the properties of the normal distribution (e.g., the mean, median, and mode are equal; the curve (and data) is symmetric about the mean)
C9D-03 Analyse and interpret the impact of alterations to data sets in each of mean, median and mode
C9D-04 Demonstrate the basic understanding of simple random sample
C9D-05 Use range, outliers, gaps, clusters to make inferences and predictions to solve problems
Probability
C9D-06 Demonstrate the knowledge of dependent and independent events, theoretical and experimental probability
C9D-07 Determine theoretical probability or experimental probability for simple and compound events

C9D-08 Differentiate between independent and dependent events
C9D-09 Determine conditional probability