BUSINESS MATHEMATICS

*Answer **Question 1** from Section **A** and **10** questions from Section **B**.*
All working, including rough work, should be done on the same sheet adjacent to the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [].

Mathematical formulae are given at the end of this question paper.
The use of calculator (fx-82/ fx-100) is allowed.

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SECTION A

(Answer **ALL** questions)

Directions: *Read the following questions carefully. For each question there are four alternatives, A, B, C and D. Choose the correct alternative and write it in the space provided.*

Question 1 **[2 × 15 = 30 marks]**

i) The co-factor of 2 in $\begin{bmatrix} 3 & 4 & 5 \\ 0 & 3 & 1 \\ 1 & 2 & 5 \end{bmatrix}$ is

A –3

B 3

C 10

D –10

Answer.....

ii) Which of the following is the differential coefficient of $e^{2x} + \cos x$ with respect to x?

A $2e^{2x} + \sin x$

B $4e^{2x} + \sin x$

C $2e^{2x} - \sin x$

D $e^{2x} - \sin x$

Answer.....

iii) Which of the following is average cost when $x = 4$, for the cost function $C(x) = 7x^2 - 4x + 8$.

A 10

B 26

C 52

D 104

Answer.....

iv) The value of x in $\begin{vmatrix} 0 & x & 1 \\ 2 & 0 & 3 \\ 4 & 1 & 5 \end{vmatrix} = 4$.

A -1

B 1

C 0

D 3

Answer.....

v) The direction cosines of a line having direction ratios 2, 3, 6 is

A $\frac{1}{3}, \frac{1}{2}, 1.$

B $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}.$

C $\frac{1}{2}, \frac{3}{8}, \frac{3}{4}.$

D $\frac{2}{12}, \frac{3}{12}, \frac{1}{12}.$

Answer.....

vi) A pair of dice is rolled. The probability of getting a sum less than 8 is

A $\frac{1}{36}$.

B $\frac{1}{18}$.

C $\frac{1}{3}$.

D $\frac{1}{2}$.

Answer.....

vii) Karchung invited 10 of his friends for a dinner. In how many ways can Karchung and his friends be seated around a round table?

A 5!

B 6!

C 9!

D 10!

Answer.....

viii) The mean deviation from the mean for the marks: 6, 10, 12, 16.

A 0

B 3

C 3.6

D 11

Answer.....

ix) $\int x e^x \, dx$

A c

B $e^x (x-1)$

C $e^x (x+1)+c$

D $e^x (x-1)+c$

Answer.....

x) Which of the following is the equation of an ellipse?

A $3x^2 - 4y^2 = 1$

B $4x^2 - 9y^2 = 36$

C $25x^2 + 4y^2 = 100$

D $x^2 + y^2 = 16$

Answer.....

xi) Which of the following is the amount of an annuity immediate of Nu 2500 in 15 years allowing interest 6% per annum.

A Nu 5819

B Nu 58190

C Nu 6168.14

D Nu 61681.40

Answer.....

xii) The gradient of the function $5x^2 + 3x + 2$ at $x = 1$ is

A 5.

B 10.

C 13.

D 15.

Answer.....

xiii) Which of the following is the range of the data 8, 10, 3, 4, 5, 10, 15?

A 7.9

B 8

C 12

D 15

Answer.....

xiv) The total revenue received from the sale of x units of a product is given by $R(x) = 7x^2 + 5x + 6$. The marginal revenue when $x = 9$ is

A 14.

B 69.

C 131.

D 618.

Answer.....

xv) Find the minimum value of $x+\frac{27}{x^3}$.

A 3

B 4

C 5

D 27

Answer.....

SECTION B

Answer any 10 questions. All questions in this section carry equal marks.
Unless otherwise stated, you may round off your answers to two decimal places.

Question 2 **[10 × 7 =70 marks]**

a) Find the co-ordinates of a point which divides internally the points (3, 7, -4), (6, 4, 2) in the ratio 1:2. **[3]**

b) Differentiate $(\cos x)^{x^2}$ with respect to x. **[4]**

Question 3

a) How many committees of 4 members each can be formed with 3 captains and 2 teachers so as to include atleast one teacher? **[3]**

b) For a new product, a manufacturer spends Nu 100,000 on the infrastructure and the variable cost is estimated as Nu 150 per unit of the product. The sale price per unit was fixed at Nu 200. Find the following **[4]**

(i) Cost Function
(ii) Revenue function
(iii) Profit function
(iv) The breakeven point

Question 4

a) Find the equation of a hyperbola whose focus is $(\pm 6, 0)$, and vertices $(\pm 2, 0)$. **[3]**

b)

i) In how many ways can the letters of the word "PROBABILITY" be arranged? **[2]**

ii) If $A=\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$, *find* A^{-1} **[2]**

Question 5

a) $\int \sin^5 x \cos x \, dx$ **[3]**

b) Karma borrowed some money and returned it in 8 equal half yearly installments of Nu 8435. What sum did he borrow if the rate of interest was 20% per annum compounded half yearly? Find also the total interest charged? **[4]**

Question 6

a) The average marks scored in English by 30 students of XII science is 60 and that of 40 students of XII Commerce is 55. Find the mean marks in English for the classes of XII science and Commerce together? **[3]**

b) Differentiate $2x^2+1$ with respect to x^4-3 **[4]**

Question 7

a) The Bhutan Telecom Limited in Mongar has 400 subscribers registered and collects fixed charges of Nu 250 per subscriber per year. The Bhutan Telecom Limited proposes to increase the annual subscription and it is believed that for every increase of Nu 1, one subscriber will discontinue the service. Find out what increase will bring maximum benefit to the Bhutan Telecom Limited. **[3]**

b) Solve the system of equations using Cramer's rule. **[4]**

$x+3y=7$

$2x+y=4$

Question 8

a) Find $\frac{dy}{dx}$ *if* $x^2 + xy + y^3 = 10$. **[3]**

b) Calculate the standard deviation for the given data. **[4]**

Tomatoes per plant	1 - 5	5 – 9	9 - 13	13 - 17	17 – 21
Number of plants	20	50	44	22	14

Question 9

a) Prove the points $A(3, 2, 0)$, $B(5, 3, 2)$ *and* $C(3, 4, 4)$ are the vertices of an isosceles triangle. **[3]**

b) Yangsel borrows Nu 50,000 from a bank on condition to repay it with compound interest at 4% per annum by annual installments of Nu 5000 each. **[4]**

(i) In how many years will the debt be paid off?

(ii) What is the value of perpetual annuity of Nu 900 a year at 3% of per annum?

Question 10

a) For the parabola $x^2 = 4 \times 3y$, find the coordinates of the focus, vertex, equation of latus rectum, equation of the directrix, the equation of axis and length of latus rectum. **[3]**

b) $\int \frac{2x+5}{(x+1)(x-3)}\,dx$ **[4]**

Question 11

a) Three cards are drawn from a full pack of cards. Find the probability that all are number cards. **[3]**

b) Using the properties of determinant express the $|A|$ in factors. **[4]**

$$|A| = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Question 12

a) Create a system of two linear equations in two variables which are inconsistent. Justify your result by using matrix method. **[3]**

b) Find the correlation co-efficient between the temperatures at two different towns on 9 different days from the table given below. Also interpret your result. **[4]**

Temp at town A	14	19	24	21	26	22	15	20	16
Temp at town B	31	36	48	37	50	45	33	41	39

Question 13

a) Chencho starts a recurring deposit at the Bank of Bhutan Limited to avail Nu 25000 at the end of 2 years. If the bank pays interest at the rate of 12% per annum compounded monthly, how much amount Chencho must deposit at the beginning of every month? **[3]**

b) For observation of pairs (x, y) of the variables $x \text{ and } y$,

$\sum x = 45,\ \ \sum y = 90,\ \ \sum x^2 = 285,\ \sum y^2 = 9240,\ \sum xy = 530,\ n = 9$ Find the; **[4]**

(i) co-efficient of regression $y \text{ on } x$.

(ii) regression equation of $y \text{ on } x$.

(iii) co-efficient of correlation.

Question 14

a) Find $\int \left(\sec^2 x - \operatorname{cosec}^2 x\right) dx$. **[3]**

b) Divide 10 into two parts so that the sum of whose squares is minimum. **[4]**

Co-ordinate Geometry

$$D=\sqrt{\left(x_2-x_2\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}$$

$$(x,y,z)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2},\frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$$

$$a_1x+b_1y+c_1z=0 \text{ and } a_2x+b_2y+c_2z=0$$

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{z}{a_1b_2-a_2b_1}$$

$$\cos\theta=\pm\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^{\,2}+b_1^{\,2}+c_1^{\,2}}\sqrt{a_2^{\,2}+b_2^{\,2}+c_2^{\,2}}}$$

Algebra

$$a^2-b^2=(a+b)(a-b)$$

$$(a\pm b)^2=a^2\pm 2ab+b^2$$

$$In\ the\ quadratic\ equation\ ax^2+bx+c=0,\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$${}^{n}p_r=\frac{n!}{(n-r)!}$$

$${}^{n}C_r=\frac{n!}{r!(n-r)!}$$

$$C_{ij}=(-1)^{i+j}M_{ij}$$

$$AA^{-1}=A^{-1}A=I$$

$$A^{-1}=\frac{1}{\det A}\cdot adjA$$

$$x=\frac{D_x}{D},y=\frac{D_y}{D},z=\frac{Dz}{D}$$

Commercial Mathematics

$$A=\frac{a}{i}(1+i)\left[(1+i)^n-1\right]$$

$$P=\frac{a}{i}\left[1-(1+i)^{-n}\right]$$

$$A(x)=\frac{C(x)}{x}$$

$$C(x)=F+V(x)$$

$$R(x)=p.x$$

$$P(x)=R(x)-C(x)$$

$$MC=\frac{d}{dx}(C(x))$$

CALCULUS

$y=x^n$, $y'=nx^{n-1}$**,**

$y=cf(x)$, $y'=cf'(x)$**,**

$$If\ y=u\pm v, then\ \frac{dy}{dx}=\frac{du}{dx}\pm\frac{dv}{dx}$$

$$If\ y=uv, then\ \frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$$

$$If\ y=\frac{u}{v},\ then\ \frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$$

Data and Probability

$$\overline{X} = \frac{\sum fx}{\sum f} \qquad or \;\; \bar{X} = \frac{\sum x}{n}$$

$$Median = L + \frac{i}{f}\left(\frac{N}{2} - c\right)$$

$$Mean\,Deviation = \frac{\sum f\left(x - \bar{x}\right)}{\sum f}$$

$$\sigma = \sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{n}} \; or \; \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$\overline{X_{12}} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$\sigma_{12} = \sqrt{\frac{n_1{\sigma_1}^2 + n_2{\sigma_2}^2 + n_1{d_1}^2 + n_2{d_2}^2}{n_1 + n_2}}$$

$$\mathrm{Cov(X, Y)} = \frac{1}{\mathrm{n}}\sum\left(\mathrm{X} - \overline{\mathrm{X}}\right)\left(\mathrm{Y} - \overline{\mathrm{Y}}\right)$$

$$r = \frac{\sum\left(x - \bar{x}\right)\left(y - \bar{y}\right)}{\sqrt{\sum\left(x - \bar{x}\right)^2\sum\left(y - \bar{y}\right)^2}} = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum\right.}}$$

$$r = \frac{\sum\left(x - \bar{x}\right)\left(y - \bar{y}\right)}{n\sigma_x\sigma_y}$$

$$b_{YX} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$b_{XY} = r\frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x\sum y}{n\sum y^2 - \left(\sum y\right)^2}$$

$$Y - \bar{Y} = \frac{\mathrm{cov}\left(X, Y\right)}{{\sigma_x}^2}\left(X - \bar{X}\right) = r\frac{\sigma_y}{\sigma_x}\left(X - \bar{X}\right)$$

$$X - \bar{X} = \frac{\mathrm{cov}\left(X, Y\right)}{{\sigma_x}^2}\left(Y - \bar{Y}\right) = r\frac{\sigma_x}{\sigma_y}\left(Y - \bar{Y}\right)$$

$$\mathrm{b}_{xy} \times \mathrm{b}_{yx} = r\frac{\sigma_x}{\sigma_y} \times r\frac{\sigma_y}{\sigma_x}$$

$$\sum y = na + b\sum x$$
$$\sum xy = a\sum x + b\sum x^2$$

$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$$
$$x - \bar{x} = b_{xy}\left(y - \bar{y}\right)$$

$$P\left(A \cup B\right) = P\left(A\right) + P\left(B\right) - P\left(A \cap B\right)$$

$$P\left(A\right) + P\left(\bar{A}\right) = 1$$

$$P\left(B/A\right) = \frac{P\left(A \cap B\right)}{P\left(A\right)}$$

$$P\left(A/B\right) = \frac{P\left(A \cap B\right)}{P\left(B\right)}$$