MATHEMATICS

*Answer **Question 1** from Section **A** and **10 questions** from Section **B**.*
All working, including rough work, should be done on the same sheet adjacent to the rest of the answers.
The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical formulae are given at the end of this question paper.
The use of calculator (Fx-82)/(Fx-100) is allowed.

SECTION A

(Answer **ALL** questions)

Directions: *Read the following questions carefully. For each question there are four alternatives, A, B, C and D. Choose the correct alternative and write it in the space provided.*

Question 1 **[2×15 = 30]**

(i) What is the sample space if two dice are rolled?

A 6

B 12

C 36

D 72

Answer……

(ii) The value of $\sin\left(2\tan^{-1}\frac{1}{3}\right)$ is

A $\frac{3}{4}$

B $\frac{3}{5}$

C $\frac{4}{5}$

D $\frac{2}{5}$

Answer……

(iii) At what value of x, the function $\frac{2}{3}x^3 + 2x^2 - 6x + 4$ is minimum?

A -8

B -3

C 1

D 8

Answer……

(iv) The conic $x^2 - 2y^2 - 2x + 8y - 1 = 0$ represent

A ellipse.

B hyperbola.

C parabola.

D straight lines.

Answer……

(v) Given that the regression coefficient of Y on $X = 0.25,\ r = 0.42$ $S.D$ of $Y = 4$. Then $S.D$ of X will be

A 6.72.

B 7.62.

C 2.37.

D 6.27.

Answer……

(vi) Which of the following is TRUE for the differential equation $x\frac{dy}{dx} + \frac{3}{dy/dx} = y^2$?

A order 1, degree 1

B order 2, degree 1

C order 1, degree 2

D order 2, degree 2

Answer……

(vii) What is the distance between the two points $(3,6,4)$ and $(-2,3,2)$?

A 5

B $\sqrt{29}$

C $\sqrt{34}$

D $\sqrt{38}$

Answer……

(viii) For what value of 'x', the points $(5,5)$, $(x,1)$ and $(11,-7)$ can be on a line?

A -14

B -7

C 7

D 14

Answer……

(ix) The conjugate of a complex number $(2-3i)^3$ is

A $-46-9i$.

B $2+3i$.

C $2-3i$.

D $-46+9i$.

Answer……

(x) Bhutan Telecom Ltd. is planning to form a 6-digit telephone numbers in Thimphu area. How many 6-digits telephone numbers that can be formed with the digits 0, 1, 2… 9, if each number starts with 32 and no digits appear more than once?

A 5040

B 1680

C 336

D 210

Answer……

(xi) If $A^3 - A^2 + A - I = 0$, then inverse of A is equal to

A $A^2 - A + I$.

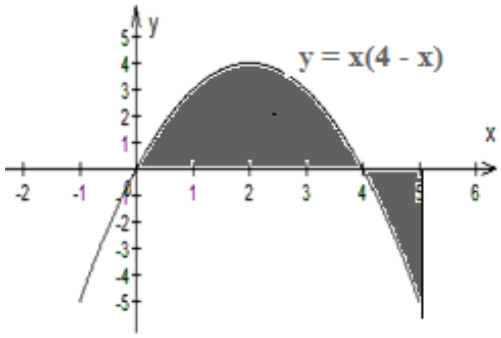
B $A^2 + A - I$.

C $A^2 + A + I$.

D $A^2 - A - I$.

Answer……

(xii) In the diagram given below, what is the area of shaded portion?



A $10\frac{2}{3}$ Sq. units

B $\frac{25}{3}$ Sq. units

C 13 Sq. units

D 25 Sq.units

Answer……

(xiii) If ω is the cube root of unity, then the value of $\left(\omega+\omega^2\right)^5$ is eqaul to

A 1.

B 0.

C ω.

D -1.

Answer……

(xiv) Which of the following is the tangent of the function $f(x)=(4x-1)^2$ at $(0,1)$?

A -8

B 9

C 16

D 24

Answer……

(xv) The angle between the pairs of straight lines represented by $x^2-5xy+4y^2=0$ is

A $\tan^{-1}\frac{3}{5}$.

B $\tan^{-1}\frac{4}{5}$.

C $\tan^{-1}\frac{5}{3}$.

D $\tan^{-1}\frac{5}{4}$.

Answer……

Section- B

Answer any 10 questions. All questions in this section carry equal marks.
Unless otherwise stated, you may round off your answers to two decimal places.

Question 2

(a) If ${}^{10}C_{r+1} = {}^{10}C_{2r}$, find the value of ${}^{7}C_{r}$ **[3]**

(b) Find the general solution of the equation $x\frac{dy}{dx} - 2y = x^2$ **[4]**

Question 3

(a) In an inter-house extempore speech competition, the judges awarded the following points to the five competitors. **[3]**

Competitors	Karma	Devika	Kezang	Arjun	Lemo
Judge I	3	7	5	10	4
Judge II	3	6	4	8	5

Calculate the Spearman's rank correlation between the judges.
Comment on the result.

(b) Solve for x: $\tan^{-1}\frac{x+1}{x-1}+\tan^{-1}\frac{x-1}{x}=\tan^{-1}(-7)$ **[4]**

Question 4

(a) Find the equation of the plane which passes through the points $A(6,2,3)$, $B(3,3,-2)$ and $C(2,-2,-1)$. **[3]**

(b) The means and standard deviations of the prices of Potato per kilogram in two towns are 20 and 15 respectively. The standard deviations of the prices are respectively 2.45 and 3.69. Create a table of values comprising **FIVE** data values of prices for respective town and verify your values to the nearest given means and standard deviations. **[4]**

Question 5

(a) Examine the equality of the statement. **[3]**

$$\sin\left[\cot^{-1}\left\{\cos\left(\tan^{-1} y\right)\right\}\right] = \sqrt{\frac{y^2+1}{y^2+2}}$$

(b) Find the vertices, foci, eccentricity and the equation of directrix of the hyperbola.
$x^2 - 4y^2 = 36$ **[4]**

Question 6

(a) Three students Yonten, Sonam and Wangmo sat for a Golden Youth Award interview. The probability of selecting them are $\frac{1}{2}, \frac{3}{4}$ and $\frac{4}{7}$ respectively. Find the probability that one of them will be selected? **[3]**

(b) Find the volume of the largest box that can be made by cutting equal squares out of the corners of the cardboard piece with a dimension of 15cm by 24cm, and then turning up the sides. **[4]**

Question 7

(a) Show that the lines joining $A(4,1,-4)$ to $B(5,0,3)$ and $C(-4,-3,-2)$ to $D(-2,-5,12)$ are parallel to each other. **[3]**

(b) Show that: $\begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ p^3 & q^3 & r^3 \end{vmatrix} = (p-q)(q-r)(r-p)(p+q+r)$ **[4]**

Question 8

(a) The region bounded by the curve $y = x(x-2)$ and the x-axis between $x = 0$ and $x = 2$ is revolved around the x-axis. Find the volume of this solid. **[3]**

(b) Use De. Moivre's Theorem to find all the values of $\left(1+i\sqrt{3}\right)^{\frac{1}{3}}$. Also find the continued product of all the values. **[4]**

Question 9

(a) Solve the following system of equation using Martin's rule. **[5]**

$4x - 2y + 3z = 1$

$x + 3y - 4z = -7$

$3x + y + 2z = 5$

(b) Find $\frac{dy}{dx}$ of $\left(x^x\right)^x$. **[2]**

Question 10

(a) Prove that the equation $3x^2 + 7xy + 2y^2 + 5x + 5y + 2 = 0$ represents a pair of straight lines. **[3]**

(b) Integrate $\frac{3x+1}{x^2-3x+2}$. **[4]**

Question 11

(a) Find the distance from the plane $x - y + z = 5$ to the point $(1, -2, 3)$ which is parallel to line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$. **[3]**

(b) Evaluate: $\int \theta^2 \cos 3\theta d\theta$ **[4]**

Question 12

(a) The directrix of a conic section is the line $x - y = 3$ and the focus is $(1, -1)$.

If the eccentricity is $\frac{1}{2}$, find the equation to the conic section. **[3]**

(b) The events A and B are such that $P(A)=\frac{2}{5}$, $P(B)=\frac{1}{4}$ and $P(A/B)=\frac{3}{5}$.

Determine the probability that

i) Neither A nor B occurs.

ii) A occurs or B occurs.

iii) B occurs if A has already occurred. **[4]**

Question 13

(a) Prove that $\left(1+x^2\right)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=0$ if $y=\tan^{-1}x$. **[3]**

(b) Find the vertex, focus, directrix and latus rectum of the given parabola.
$(y+4)^2 = 2(x+3)$. **[4]**

Question 14

(a) Evaluate: $\int \frac{\tan x}{\sec x + \tan x} dx$. **[3]**

(b) The point P represents the complex number z on an Argand diagram. Describe the locus geometrically and create an equation for the locus $|z|+|z-4|=6$. **[4]**

Trigonometry

$$\sin^{-1}x=\cos^{-1}\sqrt{1-x^2}=\tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

$$\sin^{-1}x\pm\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}\pm y\sqrt{1-x^2}\right)$$

$$\cos^{-1}x\pm\cos^{-1}y=\cos^{-1}\left(xy\mp\sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$\tan^{-1}x\pm\tan^{-1}y=\tan^{-1}\left(\frac{x\pm y}{1\mp xy}\right),xy<1$$

$$2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}=\sin^{-1}\frac{2x}{1+x^2}=\cos^{-1}\frac{1-x^2}{1+x^2}$$

$$\cos ec^{-1}x=\sin^{-1}\frac{1}{x}$$

$$\sec^{-1}x=\cos^{-1}\frac{1}{x}$$

$$\cot^{-1}x=\tan^{-1}\frac{1}{x}$$

ComplexNumbers

$$r=\sqrt{a^2+b^2}$$

$$\tan\theta=\frac{b}{a}\Rightarrow\theta=\tan^{-1}\left(\frac{b}{a}\right)$$

If $z=r\left(\cos\theta+i\sin\theta\right)$ then

$$z^n=r^n\left(\cos n\theta+i\sin n\theta\right)$$

$$z^{\frac{1}{n}}=r^{\frac{1}{n}}\left(\cos\frac{2k\pi+\theta}{n}+i\sin\frac{2k\pi+\theta}{n}\right),$$

$$k=0,1,2,3,\ldots n-1$$

Co-ordinateGeometry

$$D=\sqrt{\left(x_2-x_2\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}$$

$$(x,y,z)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2},\frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$$

$a_1x+b_1y+c_1z=0$ and $a_2x+b_2y+c_2z=0$

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{z}{a_1b_2-a_2b_1}$$

Angle between two planes,

$$\cos\theta=\pm\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

distance of a point from a plane $=\pm\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}$

$$\left(x,y\right)=\left(\frac{m_1x_1+m_2x_2}{m_1+m_2},\frac{m_1y_1+m_2y_2}{m_1+m_2}\right)$$

$Angle\,between\,the\,lines ax^2+2hxy+by^2=0,$

$$\tan\theta=\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$$

$equation\,of\ bi\sec tor,\frac{x^2-y^2}{a-b}=\frac{xy}{h}$

points of intersection, $\left(\frac{hf-bg}{ab-h^2},\frac{gh-af}{ab-h^2}\right)$

Algebra

$a^2 - b^2 = (a+b)(a-b)$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

In the quadratic equation $ax^2 + bx + c = 0,\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$${}^{n}p_r = \frac{n!}{(n-r)!}$$

$${}^{n}C_r = \frac{n!}{r!(n-r)!}$$

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$A A^{-1} = A^{-1} A = I$$

$$A^{-1} = \frac{1}{\det A} \cdot adjA$$

$$x = \frac{D_x}{D}, y = \frac{D_y}{D}, z = \frac{Dz}{D}$$

$$1 + 2 + 3 + ... + (n-1) = \frac{1}{2} n(n-1)$$

$$1^2 + 2^2 + 3^2 + ... + (n-1)^2 = \frac{1}{6} n(n-1)(2n-1)$$

$$1^3 + 2^3 + 3^3 + ... + (n-1)^3 = \left\{ \frac{n(n-1)}{2} \right\}^2$$

CALCULUS

$y = x^n,\ y' = nx^{n-1}$,

$y = cf(x),\ y' = cf'(x)$,

If $y = u \pm v$, *then* $\dfrac{dy}{dx} = \dfrac{du}{dx} \pm \dfrac{dv}{dx}$

If $y = uv$, *then* $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

If $y = \dfrac{u}{v}$, *then* $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int uv\,dx = u\int v dx - \int \left(\frac{du}{dx} \int v dx \right) dx.$$

$$\int_a^b f(x)dx = \lim_{h \to 0} h \left[\sum_{r=0}^{n-1} f(a + rh) \right]$$

$$\frac{dy}{dx} + py = Q, I.F = ye^{\int p dx},$$

general solution, $y.IF = \int (Q.IF)dx + c$

$$V = \pi \int_a^b y^2 dx \quad A = \int_a^b y dx$$

Volume of Cone $= \dfrac{1}{3}\pi r^2 h$

Volume of Sphere $= \dfrac{4}{3}\pi r^3 h$

Volume of Cylinder $= \pi r^2 h$

S.Area of Cone $= \pi rl + \pi r^2$

S.Area of Sphere $= 4\pi r^2$

S.Area of Cylinder $= 2\pi rh + 2\pi r^2$

Data and Probability

$$\overline{X} = \frac{\sum fx}{\sum f} \qquad or \quad \bar{X} = \frac{\sum x}{n}$$

$$Median = L + \frac{i}{f}\left(\frac{N}{2} - c\right)$$

$$\sigma = \sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{n}} \; or \; \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$\overline{X} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$Mean\,Deviation = \frac{\sum f\left(x - \bar{x}\right)}{\sum f}$$

$$\sigma_{12} = \sqrt{\frac{n_1{\sigma_1}^2 + n_2{\sigma_2}^2 + n_1{d_1}^2 + n_2{d_2}^2}{n_1 + n_2}}$$

$$\mathrm{Cov(X,Y)} = \frac{1}{\mathrm{n}}\sum\left(\mathrm{X} - \overline{\mathrm{X}}\right)\left(\mathrm{Y} - \overline{\mathrm{Y}}\right)$$

$$r = \frac{\sum\left(x - \bar{x}\right)\left(y - \bar{y}\right)}{\sqrt{\sum\left(x - \bar{x}\right)^2\sum\left(y - \bar{y}\right)^2}} = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

$$r = \frac{\sum\left(x - \bar{x}\right)\left(y - \bar{y}\right)}{n\sigma_x\,\sigma_y}$$

$$r = 1 - \frac{6\sum d^2}{n(n^2 - 1)}, \quad Correction\ factor = \frac{1}{12}\left(m^3 - m\right)$$

$$r = \pm\sqrt{b_{xy}.b_{yx}}$$

$$b_{YX} = r\,\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$b_{XY} = r\,\frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x\sum y}{n\sum y^2 - \left(\sum y\right)^2}$$

$$Y - \bar{Y} = \frac{\mathrm{cov}\left(X,Y\right)}{{\sigma_x}^2}\left(X - \bar{X}\right) = r\,\frac{\sigma_y}{\sigma_x}\left(X - \bar{X}\right)$$

$$X - \bar{X} = \frac{\mathrm{cov}\left(X,Y\right)}{{\sigma_x}^2}\left(Y - \bar{Y}\right) = r\,\frac{\sigma_x}{\sigma_y}\left(Y - \bar{Y}\right)$$

$$\mathrm{b}_{xy} \times \mathrm{b}_{yx} = r\,\frac{\sigma_x}{\sigma_y} \times r\,\frac{\sigma_y}{\sigma_x}$$

$$\sum y = na + b\sum x$$
$$\sum xy = a\sum x + b\sum x^2$$

$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$$
$$x - \bar{x} = b_{xy}\left(y - \bar{y}\right)$$

$$P\left(A \cup B\right) = P\left(A\right) + P\left(B\right) - P\left(A \cap B\right)$$

$$P\left(A\right) + P\left(\bar{A}\right) = 1$$

$$P\left(B/A\right) = \frac{P\left(A \cap B\right)}{P\left(A\right)}$$

$$P\left(A/B\right) = \frac{P\left(A \cap B\right)}{P\left(B\right)}$$