## BUSINESS MATHEMATICS

*Answer **Question 1** from Section **A** and **10** questions from Section **B**.*
*All working, including rough work, should be done on the same sheet adjacent to the rest of the answer.*
*The intended marks for questions or parts of questions are given in brackets [].*

***Mathematical formulae are given at the end of this question paper.***
***The use of calculator (fx-82/ fx-100) is allowed.***

### SECTION A

(Answer **ALL** questions)

**Direction:** *For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than **ONE** alternative. If there are more than one choice circled, **NO** score will be awarded.*

**Question 1** **[2×15 = 30]**

i. The value of ${}^{n}c_{n}$ is

**A** 0.

**B** 1.

**C** 2.

**D** $n$.

ii. Study the following table.

| Functions | Derivatives |
|---|---|
| $x^2$ | $2x$ |
| $x^2+2x$ | $2x+2$ |
| $3x^2+4x+2$ | $6x+4$ |

What is the derivative of $2x^3+4x^2-x+3$ ?

**A** $\frac{x^4}{2}+\frac{4x^3}{3}-\frac{x^2}{2}+3x$

**B** $6x^2+8x+1$

**C** $6x^2-8x+1$

**D** $6x^2+8x-1$

iii. If an amount is deposited at 5% p.a. so that it gives a return of Nu 450 p.a. in perpetuity. What is the present worth?

**A** Nu 9000

**B** Nu 2250

**C** Nu 900

**D** Nu 90

iv. For conic section, the constant ratio is called eccentricity. Depending on the value of eccentricity, are classified into three types. If the values of eccentricity are $\sqrt{2}, \frac{\sqrt{2}}{\sqrt{2}}, \frac{1}{\sqrt{2}}$, these represent

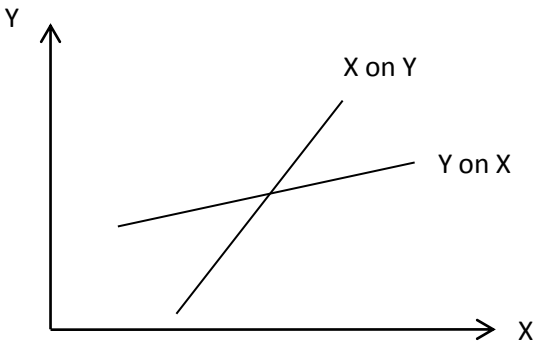
**A** ellipse, hyperbola and parabola respectively.

**B** parabola, ellipse and hyperbola respectively.

**C** hyperbola, parabola and ellipse respectively.

**D** hyperbola, ellipse and parabola respectively.

v.




The above graph represents

**A** low degree positive correlation.

**B** low degree negative correlation.

**C** high degree positive correlation.

**D** high degree negative correlation.

vi. Let A be $(3,4,5)$ and B be $(-2,-1,0)$. What will be the coordinates of the point which is two-fifth of the way from A to B?

**A** $\frac{1}{2},\frac{3}{2},\frac{5}{2}$

**B** $0,1,2$

**C** $1,2,3$

**D** $3,2,1$

vii. $\int x\sin x dx$ is

**A** $x\cos x+\sin x+c$.

**B** $-x\cos x+\sin x+c$.

**C** $x\cos x-\sin x+c$.

**D** $-x\cos x-\sin x+c$.

viii. The demand function of the monopolist is given by $p=150-3x-x^2$. Find the marginal revenue when $x=5$.

**A** –45

**B** –13

**C** 45

**D** 110

ix. A and B are singular matrix and non-singular matrix respectively.From the given matrices, find the value of $x$.

$A=\begin{bmatrix} 2 & 3 \\ 4 & x \end{bmatrix}$ and $B=\begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}$

**A** 6

**B** 8

**C** $x+8$

**D** $x-8$

x. For a set of ungrouped values, the following sums are found $n=15, \sum x=480, \sum x^2=15735$, Find the standard deviation.

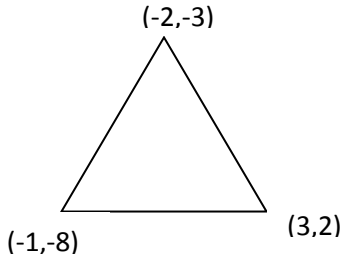
**A** $-5$

**B** $-4$

**C** 4

**D** 5

xi.




Find the area of the given figure.

**A** 30 sq unit

**B** 15 sq unit

**C** $-15$ sq unit

**D** $-30$ sq unit

xii. On 17 December, Tashi won lottery and saves money so that she can pay her loan every first week of January. What is the amount of annuity of Nu 100 payable yearly for 15 years at 10% compounded yearly?

**A** Nu 418

**B** Nu 1000

**C** Nu 3177

**D** Nu 3495

xiii. The minimum value of the function $\left(x+\frac{4}{x^2}\right)$ is

**A** 0.

**B** 2.

**C** 3.

**D** infinite.

xiv. The probability that Dorji will receive C and D grade in mathematics are as follows:

| | |
|---|---|
| C | 0.28 |
| D | 0.61 |

What is the probability that Dorji will receive C or D in Mathematics?

**A** 0.89

**B** 0.61

**C** 0.33

**D** 0.28

xv. What is the gradient of the graph $y=\log x$ at (1,0)?

**A** 0

**B** 1

**C** 2

**D** 3

## SECTION B [70 Marks]

*Answer **any** 10 questions. All questions in this section have equal marks.*

**Question 2**

a) The average weight of 30 girls is 45 kg and that of 25 boys is 51 kg in a class. Find the mean weight in kg for the entire class. **[3]**

b) $\int \frac{2x+5}{x^2-x-2}\,dx$ **[4]**

**Question 3**

a) Find foci and eccentricity of the hyperbola, whose equation is $\frac{x^2}{64} - \frac{y^2}{36} = 1$. **[3]**

b) Use matrix method to solve the system of equation.

$4x - 3y = 11, \qquad 3x + 7y = -1$ **[4]**

**Question 4**

a) The cost function is given by $C(x) = 5x^2 - 3x - 14$. Find the average cost when $x = 7$ and $x = 3.5$ . **[3]**

b) Find the derivative of the function $x^{x^x}$ with respect to $x$. **[4]**

**Question 5**

a) Using the properties of determinant, find the factors of [4]

$$\begin{bmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{bmatrix}.$$

b) A letter is chosen at random from the letters of the word 'ELEPHANT'.
What is the probability that letters will be **[3]**

i) H.

ii) A vowel.

iii) H or a vowel.

**Question 6**

a) How many eight digit mobile number can be formed with the digit 0,1,2...8,9, if each number starts with 17 and no digit appears more than once? **[3]**

b) Compute the Karl Pearson's coefficient of correlation of the given table. **[4]**

| | | | | | | |
|---|---|---|---|---|---|---|
| X | 2 | 4 | 5 | 6 | 8 | 11 |
| Y | 18 | 12 | 10 | 8 | 7 | 5 |

**Question 7**

a) Karma Steel Private Limited has fixed cost of Nu 15,000 and cost of producing one unit of its product is Nu 60. If each unit sells for Nu 80. Find the break even value and also the profit function. **[4]**

b) Sum of two numbers is 20 and whose product is as large as possible. Find two positive numbers. **[3]**

**Question 8**

a) A, B, C are the points $(1,4,2),(-2,1,2),(2,-3,4)$. Find the angles of the triangle ABC using direction ratios. **[4]**

b) Using determinant, solve the system of equations. **[3]**

$2x + y = 1$ and $x - 2y = 8$

**Question 9**

a) Find the least number of years for which an annuity of Nu 10,000 must run in order that its amount just exceeds Nu 160,000 at 5% p.a. compounded annually. **[3]**

b) Calculate the rank correlation coefficient for the marks obtained by 11 students in two subjects as given below. **[4]**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Maths | 24 | 29 | 19 | 14 | 30 | 19 | 27 | 30 | 20 | 28 | 11 |
| Physics | 37 | 35 | 16 | 26 | 23 | 27 | 19 | 20 | 16 | 11 | 21 |

**Question 10**

a) Differentiate $\sin xy + \frac{x}{y} = x^2 - y.$ **[3]**

b) If $C = 2x\left(\frac{x+4}{x+1}\right)+6$ is the total cost of production, show that the marginal cost falls continuously as the output $x$ increases. **[4]**

**Question 11**

a) Find the equation of ellipse whose eccentricity is $\frac{1}{2}$ and whose foci are at the point $(\pm 2,0)$. **[3]**

b) $\int \frac{dx}{\sin(x-a)\cos(x-b)}$. **[4]**

**Question 12**

a) Means and standard deviations of two distributions of 100 and 150 items are 50, 7 and 40, 8 respectively. Find the mean and standard deviation of all the 250 items taken together. **[4]**

b) Differentiate $\left(\log_e x\right)\left(\log_a x\right)$. **[3]**

**Question 13**

a) Gyeltshen has been raising a fund at 8% compounded annually, which will provide him with an annual income of Nu 60,000 for 15 years. The first payment has been made on his 60th birthday. If he wishes to reduce the number of payments to 10, what will be his annual income? **[4]**

b) $\int \frac{x^3}{(x^2+1)^3}\,dx$ **[3]**

**Question 14**

**a)** Derive the equation of the locus of a point equidistant from the points (1,-2,3) and (-3,4,2). **[3]**

b) In a school archery team, there are 16 players of whom 4 are teachers. In how many ways can a team of 11 players be selected so as to include

i) Only one teacher? **[1]**

ii) At least one teacher? **[3]**

## MATHEMATICS FORMULAE

**Co-ordinate Geometry**

$$(x, y, z) = \left( \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}, \frac{m_1 z_2 + m_2 z_1}{m_1 + m_2} \right)$$

$$D = \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

$$\cos\theta = \pm \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

**Algebra**

$a^2 - b^2 = (a+b)(a-b)$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

*In the quadratic equation* $ax^2 + bx + c = 0,\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$${}^n p_r = \frac{n!}{(n-r)!}$$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$A A^{-1} = A^{-1}A = I$$

$$A^{-1} = \frac{1}{\det A} \cdot adjA$$

$$x = \frac{D_x}{D}, y = \frac{D_y}{D}, z = \frac{Dz}{D}$$

**Commercial Mathematics**

$$A = \frac{a}{i}(1+i)\left[(1+i)^n - 1\right]$$

**Data and Probability**

$$P = \frac{a}{i}\left[1 - (1+i)^{-n}\right]$$

$$A(x) = \frac{C(x)}{x},\ M(x) = \frac{d}{dx}(C(x))$$

$$C(x) = F + V(x)$$

$$R(x) = xG(x)$$

$$P(x) = R(x) - C(x)$$

$$MC = \frac{d}{dx}(C(x))$$

***CALCULUS***

$y = x^n,\ y' = nx^{n-1}$,

$$If\ y = u \pm v, then\ \frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$If\ y = uv, then\ \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$If\ y = \frac{u}{v},\ then\ \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int uv\,dx = u\int v dx - \int\left(\frac{du}{dx}\int v dx\right)dx.$$

$$\overline{X} = \frac{\sum fx}{\sum f} \quad or\ \overline{X} = \frac{\sum x}{n}$$

$$Median = L + \frac{i}{f}\left(\frac{N}{2} - c\right)$$

$$\sigma = \sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{n}} \; or \; \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$\overline{X_{12}} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$\sigma_{12} = \sqrt{\frac{n_1{\sigma_1}^2 + n_2{\sigma_2}^2 + n_1{d_1}^2 + n_2{d_2}^2}{n_1 + n_2}}$$

$$\mathrm{Cov(X, Y)} = \frac{1}{\mathrm{n}}\sum\left(\mathrm{X} - \overline{\mathrm{X}}\right)\left(\mathrm{Y} - \overline{\mathrm{Y}}\right)$$

$$r = \frac{\sum\left(x - \bar{x}\right)\left(y - \bar{y}\right)}{\sqrt{\sum\left(x - \bar{x}\right)^2 \sum\left(y - \bar{y}\right)^2}} = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

$$r = \frac{\sum\left(x - \bar{x}\right)\left(y - \bar{y}\right)}{n\sigma_x\,\sigma_y}$$

$$r = 1 - \frac{6\sum d^2}{n(n^2 - 1)},$$

$$Correction\ factor = \frac{1}{12}\left(m^3 - m\right)$$

$$r = \pm\sqrt{b_{xy}.b_{yx}}$$

$$b_{YX} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$b_{XY} = r\frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - \left(\sum y\right)^2}$$

$$Y - \bar{Y} = \frac{\mathrm{cov}\left(X, Y\right)}{{\sigma_x}^2}\left(X - \bar{X}\right) = r\frac{\sigma_y}{\sigma_x}\left(X - \bar{X}\right)$$

$$X - \bar{X} = \frac{\mathrm{cov}\left(X, Y\right)}{{\sigma_x}^2}\left(Y - \bar{Y}\right) = r\frac{\sigma_x}{\sigma_y}\left(Y - \bar{Y}\right)$$

$$\mathrm{b}_{xy} \times \mathrm{b}_{yx} = r\frac{\sigma_x}{\sigma_y} \times r\frac{\sigma_y}{\sigma_x}$$

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$$

$$x - \bar{x} = b_{xy}\left(y - \bar{y}\right)$$

$$P\left(A \cup B\right) = P\left(A\right) + P\left(B\right) - P\left(A \cap B\right)$$

$$P\left(A\right) + P\left(\overline{A}\right) = 1$$

$$P\left(B/A\right) = \frac{P\left(A \cap B\right)}{P\left(A\right)}$$

$$P\left(A/B\right) = \frac{P\left(A \cap B\right)}{P\left(B\right)}$$