MATHEMATICS

*Answer **Question 1** from Section **A** and **10** questions from Section **B**.*
All working, including rough work, should be done on the same sheet adjacent to the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [].

Mathematical formulae are given at the end of this question paper.
The use of calculator (fx-82/ fx-100) is allowed.

SECTION A

(Answer **ALL** questions)

Direction: *For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than **ONE** alternative. If there are more than onechoice circled, **NO** score will be awarded.*

Question 1 **[2×15 = 30]**

i. If the principal value of $y = \cot^{-1}\sqrt{3}$ is $\frac{\pi}{6}$, then the domain of y is

A Imaginary Numbers.

B Real Numbers.

C Natural Numbers.

D Rational Numbers.

ii. The point of intersection of the pair of lines $20x^2 - 20xy + 7y^2 = 0$ is

A $(0,1)$.

B $(-2,-1)$.

C $(-1,2)$.

D $(0, 0)$.

iii. If $\Delta = \begin{vmatrix} x+2 & 3 & 3 \\ 3 & x+4 & 5 \\ 3 & 5 & x+4 \end{vmatrix} = 0$, then one of the factors of Δ is

A x -1.

B $x-2$.

C $x-3$.

D $x-4$.

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iv. At what point $y = x^3 - 3x^2 + 3x$ is in flexional?

A $(1,0)$

B $(2,1)$

C $(1,1)$

D $(3,2)$

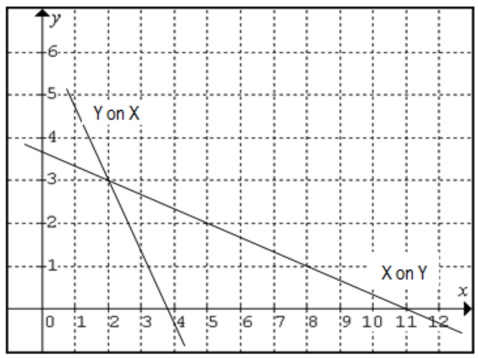
v. The following graph shows the regression equations of X and Y series. What are the arithmetic means of these two series?

A 2,3

B 3, 4

C 4, 5

D 5, 6



vi. A democracy club of 10 members is to be selected amongst 9 boys and 6 girls. In how many ways can the club in-charge do the selection so as to include at least 4 girls?

A 358

B 80424

C 2142

D 540

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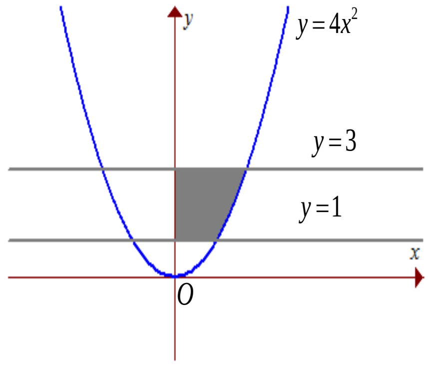
vii. The area of the shaded region is



A $\frac{1}{3}\left(2\sqrt{2}-8\right)$ Sq. Units .

B $\frac{1}{3}\left(3\sqrt{3}-1\right)$ Sq. Units .

C $\frac{1}{3}\left(3\sqrt{2}-5\right)$ Sq. Units .

D $\frac{1}{3}\left(2\sqrt{3}-9\right)$ Sq. Units .

viii. For what value of x will the line through $(1,2,3)$ and $(2,1,4)$ be perpendicular to the line through $(3,x,5)$ and $(6,6,4)$?

A 1

B 2

C 3

D 4

ix. It is given that the events A and B are such that $P(A)=\frac{1}{4}, P\left(A/B\right)=\frac{1}{2}$ and $P\left(B/A\right)=\frac{2}{3}$. Then $P(B)$ is

A $\frac{1}{3}$.

B $\frac{1}{2}$.

C $\frac{2}{3}$.

D $\frac{5}{6}$.

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x. The multiplication of two complex numbers $Z_1 = x_1 + y_1 i$ and $Z_2 = x_2 + y_2 i$ is defined as $Z_1 Z_2 = x_1 x_2 - y_1 y_2 + (x_1 y_2 + x_2 y_1) i$ .What is the argument of $Z_1 Z_2$ if

$Z_1 = \frac{\sqrt{3}}{2} + \frac{i}{2}, \qquad Z_2 = \frac{\sqrt{3}}{2} - \frac{i}{2}$?

A 30^0

B 0^0

C 45^0

D 60^0

xi. Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$.The only correct statement about the matrix A is

A A^{-1} does not exist.

B $A = (-1) I$, When I is a unit matrix.

C A is a zero matrix.

D $A^2 = I$.

xii. The general solution of $\frac{dy}{dx} + \frac{y}{x} = x^2$ is $xy = \frac{x^4}{4} + c$ .What is the particular solution of $\frac{dy}{dx} = \frac{y}{x}, y(1) = 1$?

A $x^2 - y = 0$

B $x - y = 0$

C $y - \log x = 1$

D $xy - 1 = 0$

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xiii. Twenty meters of wire is available to fence a flower bed in the form of a rectangle. If the flower bed has maximum area, then its dimension is

A 5×5.

B 5×4.

C 4×4.

D 6×5.

xiv. The following table shows the equation of parabolas and its length of latus rectum

Equation	**Latus Rectum**
$y^2 = 18x$	18
$x^2 = 10y$	10
$4(x-1)^2 = -7(y-3)$	…………………………

Complete the table.

A $-\frac{4}{7}$

B $-\frac{7}{4}$

C $\frac{7}{4}$

D 7

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xv. The direction cosines of the normal to the plane $x+2y-2z=9$ are listed below:

I $\frac{1}{3}$

II $\frac{2}{3}, \frac{1}{3}$

III $\frac{2}{3}$

IV $-\frac{2}{3}$

What are the direction cosines along *x* and *z* axis?

A III and IV

B I and IV

C II only

D I and III

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SECTION B [70 Marks]

*Answer **any** 10 questions. All questions in this section have equal marks.*

Question 2

a) The value of a determinant can be found using expansion method or without expansion. If $|A| = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$, give the factors of $|A|$ without expansion. **[4]**

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b) Find $\frac{dy}{dx}$ if $y=\sqrt{\frac{1-\cos x}{1+\cos x}}$ **[3]**

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Question 3

a) If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$, prove that $a + b + c = abc$. **[4]**

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b) G20 conference is to be held among prime ministers of 20 countries. In how many ways can they be seated in a round table if 3 particular prime ministers are

(i) always together.

(ii) never together. **[3]**

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Question 4

a) Evaluate: $\int (\log x)^2 \ dx$ **[4]**

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b) Sangay calculates the eccentricity of the curve $\frac{x^2}{4}+\frac{y^2}{36}=1$ is 2. Do you agree with him? Explain. What is the eccentricity of the curve? Also find its foci. **[3]**

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Question 5

a) Solve for x and y if $x + yi - (7 + 4i) = 3 - 5i$. **[2]**

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b) If $A=\begin{bmatrix} 8 & -4 & 1 \\ 10 & 0 & 6 \\ 8 & 1 & 6 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations: $8x-4y+z=5$, $10x+6z=4$, $8x+y+6z=\frac{5}{2}$ **[5]**

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Question 6

a) Find the possible value(s) of x if $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$. **[3]**

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b) Compute the distance of the point $\left(1,-2,3\right)$ from the plane $x-y+z=5$ measured along a line parallel to $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$. **[4]**

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Question 7

a) Find the largest possible area of a right angled triangle whose hypotenuse is 6 cm long. **[4]**

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b) Find the equation of the bisectors of the angles between the pair of straight lines represented by the equation $x^2 - 2xy - 2y^2 = 0$. Also find the angle between them. **[3]**

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Question 8

a) The expression $\frac{x^2+1}{x^2-1}$ may also be written as $1+2h$ where $h=\frac{1}{x^2-1}$.

Use this to evaluate $\int(1+2h)\,dx$. **[4]**

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b) Find the equation of the plane through $(2,3,4)$ and $(1,1,-3)$, and parallel to y – axis. **[3]**

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Question 9

a) Calculate the area of the region bounded by the curve $y = 2x - x^2$ and the line $y = x$. The given points of intersection of two curves are $(0,0)$ and $(1,1)$.
If the region bounded by the curves is rotated through four right angle about the x – axis, find the volume of the solid so formed. **[4]**

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b) Lines ***OA, OB*** are drawn from ***O*** with direction cosines proportional to $(-2,4,7)$ and $(1,-1,-2)$ respectively. Find the direction cosines of the normal to the plane ***AOB***. **[3]**

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Question 10

a) Tandin and Lhaden appear in an interview for two vacancies in the same post. The probability of Tandin's selection is $\frac{1}{7}$ and that of Lhaden's selection is $\frac{1}{5}$. **[4]**

(i) What is the probability that both of them will be selected?

(ii) What is the probability that at least one of them will be selected?

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b) Show that $4x^2 + 4xy + y^2 + 6x + 3y + 1 = 0$ represents a pair of straight lines. **[3]**

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Question 11

a) Find the equation of the parabola whose focus is $(-2,1)$ and directrix is $y-x=4$. Find also the length of a latus rectum. **[4]**

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b) $y = \sqrt{3x+2}$, prove that $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$. **[3]**

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Question 12

a) Find all the values of $\left(\sqrt{3}+i\right)^{2/3}$. Also find the continued product of the three values. **[4]**

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b) Solve:

if $\frac{dy}{dx} + y\sec x = \tan x$ **[3]**

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Question 13

a) Find the solution of $\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$ when $x = 1, y = \frac{\pi}{2}$. **[3]**

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b) Compute the standard deviation, coefficient of variation from the marks obtained by ten students:

50	55	57	49	54	61	64	59	59	56

How would the result be affected if it is decided to increase the marks for each of the above students by 5 marks? **[4]**

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Question 14

a) Find the most likely price in Thimphu (x) corresponding to the price Nu 80 at Trashigang (y) from the following data: **[4]**

$$r = 0.7$$

	Thimphu	Trashigang
Average Price	70	65
Standard Deviation	4	3.5

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b) The general equation of circle is $x^2+y^2+2gx+2fy+d=0$ and its centre and radius are defined as $(-g,-f)$ and $\sqrt{g^2+f^2-d}$ respectively.
Use this to illustrate in the complex plane set of points Z satisfying $|Z-4|<1$. **[3]**

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MATHEMATICS FORMULAE

Trigonometry

$$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

$$\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right)$$

$$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left(\frac{x \pm y}{1 \mp xy}\right), xy < 1$$

$$2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2} = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}$$

$$\operatorname{cosec}^{-1} x = \sin^{-1}\frac{1}{x}$$

$$\sec^{-1} x = \cos^{-1}\frac{1}{x}$$

$$\cot^{-1} x = \tan^{-1}\frac{1}{x}$$

Complex Numbers

$$r = \sqrt{a^2 + b^2}$$

$$\tan\theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

If $z = r(\cos\theta + i\sin\theta)$ then

$$z^n = r^n(\cos n\theta + i\sin n\theta)$$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{2k\pi+\theta}{n} + i\sin\frac{2k\pi+\theta}{n}\right),$$

$$k = 0, 1, 2, 3, \ldots n-1$$

Co–ordinate Geometry

$$D = \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$(x, y, z) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$$

$a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

Angle between two planes,

$$\cos\theta = \pm\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

distance of a point (x_1, y_1, z_1) *from a plane*

$$= \pm\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$$

distance of a point (x_1, y_1) *from a line*

$$= \pm\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$$

$$(x, y) = \left(\frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \frac{m_1y_1 + m_2y_2}{m_1 + m_2}\right)$$

Angle between the lines $ax^2 + 2hxy + by^2 = 0$,

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

equation of bisector, $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

points of intersection, $\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$

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Algebra

$${}^{n}p_{r} = \frac{n!}{(n-r)!}$$

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$AA^{-1} = A^{-1}A = I$$

$$A^{-1} = \frac{1}{\det A} \cdot adjA$$

$$x = \frac{D_x}{D}, y = \frac{D_y}{D}, z = \frac{Dz}{D}$$

$$1+2+3+.....+(n-1) = \frac{1}{2}n(n-1)$$

$$1^2+2^2+3^2+......+(n-1)^2 = \frac{1}{6}n(n-1)(2n-1)$$

$$1^3+2^3+3^3+......+(n-1)^3 = \left\{\frac{n(n-1)}{2}\right\}^2$$

CALCULUS

$y = x^n$, $y' = nx^{n-1}$,

$y = cf(x)$, $y' = cf'(x)$,

$$If\ y = u \pm v, then \frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$If\ y = uv, then \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$If\ y = \frac{u}{v}, then \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int uv\,dx = u\int vdx - \int\left(\frac{du}{dx}\int vdx\right)dx.$$

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[\sum_{r=0}^{n-1} f(a+rh)\right]$$

$$\frac{dy}{dx} + py = Q, I.F = ye^{\int pdx},$$

$$general\ solution, y.IF = \int (Q.IF)dx + c$$

$$V = \pi\int_a^b y^2dx \quad A = \int_a^b ydx$$

$$Volume\ of\ Cone = \frac{1}{3}\pi r^2 h$$

$$Volume\ of\ Sphere = \frac{4}{3}\pi r^3 h$$

$$Volume\ of\ Cylinder = \pi r^2 h$$

$$S.Area\ of\ Cone = \pi rl + \pi r^2$$

$$S.Area\ of\ Sphere = 4\pi r^2$$

$$S.Area\ of\ Cylinder = 2\pi rh + 2\pi r^2$$

Data and Probability

$$\overline{X} = \frac{\sum fx}{\sum f} \quad or \quad \overline{X} = \frac{\sum x}{n}$$

$$Median = L + \frac{i}{f}\left(\frac{N}{2} - c\right)$$

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$$\sigma = \sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{n}} \quad or \quad \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$\overline{X} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$Mean\,Deviation = \frac{\sum f\left(x - \bar{x}\right)}{\sum f}$$

$$\sigma_{12} = \sqrt{\frac{n_1\sigma_1{}^2 + n_2\sigma_2{}^2 + n_1d_1{}^2 + n_2d_2{}^2}{n_1 + n_2}}$$

$$\text{Cov}(X, Y) = \frac{1}{n}\sum\left(X - \overline{X}\right)\left(Y - \overline{Y}\right)$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2\sum(y-\bar{y})^2}} = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{n\sigma_x\,\sigma_y}$$

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}, \quad Correction\ factor = \frac{1}{12}\left(m^3 - m\right)$$

$$r = \pm\sqrt{b_{xy}.b_{yx}}$$

$$b_{YX} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$b_{XY} = r\frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x\sum y}{n\sum y^2 - \left(\sum y\right)^2}$$

$$Y - \overline{Y} = \frac{\text{cov}(X,Y)}{\sigma_x{}^2}\left(X - \overline{X}\right) = r\frac{\sigma_y}{\sigma_x}\left(X - \overline{X}\right)$$

$$X - \overline{X} = \frac{\text{cov}(X,Y)}{\sigma_x{}^2}\left(Y - \overline{Y}\right) = r\frac{\sigma_x}{\sigma_y}\left(Y - \overline{Y}\right)$$

$$\text{b}_{xy} \times \text{b}_{yx} = r\frac{\sigma_x}{\sigma_y} \times r\frac{\sigma_y}{\sigma_x}$$

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$$

$$x - \bar{x} = b_{xy}\left(y - \bar{y}\right)$$

$$P\left(A \cup B\right) = P\left(A\right) + P\left(B\right) - P\left(A \cap B\right)$$

$$P\left(A\right) + P\left(\overline{A}\right) = 1$$

$$P\left(B/A\right) = \frac{P\left(A \cap B\right)}{P\left(A\right)}$$

$$P\left(A/B\right) = \frac{P\left(A \cap B\right)}{P\left(B\right)}$$

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