SECTION A

(Answer **ALL** questions)

Direction: *For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than **ONE** alternative. If there are more than one choice circled, **NO** score will be awarded.*

Question 1 **[2×15 = 30]**

i) In how many ways can the letters of the word "COMBINE" be arranged?

A 720

B 900

C 5040

D 6080

ii) Chimi picked up two numbers from a box and added them to a total of 24. What could be the two numbers that she picked, if their product was as large as possible?

A (10, 14)

B (9, 15)

C (13, 11)

D (12, 12)

iii) If the total cost function of a firm is given by $C(x) = 3x^2 - 6x + 5$, then its average cost function would be

A $3x - 6 + \frac{5}{x}.$

B $-3x + 6 + \frac{5}{x}.$

C $3x - 6 - \frac{5}{x}.$

D $-3x + 6 - \frac{5}{x}.$

iv) There are two lines z_1 and z_2. The direction cosines of the line z_1 are $\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}$ and the direction cosines of the line z_2 are $\frac{\sqrt{3}}{4}, x, \frac{-\sqrt{3}}{2}$. The angle between the two lines is 120^o. Find the value of x.

A $-\frac{1}{4}$

B $-\frac{1}{2}$

C $\frac{1}{4}$

D $\frac{1}{2}$

v) A bag contains 2 white marbles, 4 blue marbles and 6 red marbles. A marble is drawn at random from the bag. What is the probability that it is NOT a blue marble?

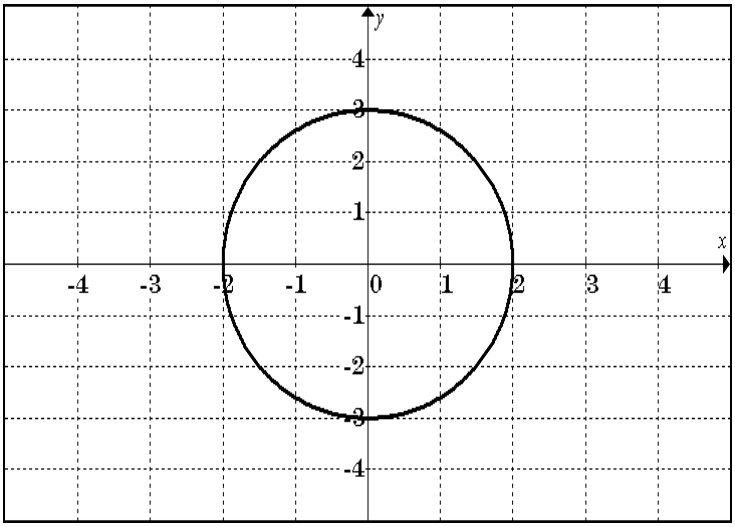
A $\frac{1}{6}$

B $\frac{1}{6}$

C $\frac{1}{2}$

D $\frac{2}{3}$

vi) The figure given below represents a vertical ellipse.



Which of the following is the equation for the above figure?

A $\frac{x^2}{4} - \frac{y^2}{9} = 1$

B $\frac{x^2}{9} - \frac{y^2}{4} = 1$

C $\frac{x^2}{4} + \frac{y^2}{9} = 1$

D $\frac{x^2}{9} + \frac{y^2}{4} = 1$

vii) $\int \frac{\cos 5x}{2}\, dx$ is equal to

A $-\frac{\sin 5x}{10} + C.$

B $\frac{\sin 5x}{10} + C.$

C $\frac{\sin 5x}{2} + C.$

D $-\frac{\sin 5x}{2} + C.$

viii) Passang decides to deposit a certain amount of money at the end of each year in a bank which pays 3 % p.a. as compound interest. If his accumulation at the end of 15 years is Nu 55,800, what is his yearly deposit?

A Nu 3000

B Nu 2913

C Nu 2500

D Nu 1925

ix) What is the value of x in the determinant $\begin{vmatrix} 0 & 2 & x \\ -1 & 8 & 3 \\ 0 & 5 & 1 \end{vmatrix} = 7$?

A $-\frac{9}{5}$

B -1

C 1

D $\frac{9}{5}$

x) The number of students in Class X A and X B are 30 and 35 respectively. The mean scores of students in a Mathematics test are as follows:

X A	**X B**	**X A and X B**
70	?	62

Find the mean score of students of Class X B.

A 24.31

B 34.21

C 55.14

D 65.21

xi) If $A=\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ *and* $B=\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$, then the value of AB' is

A $\begin{bmatrix} 8 & 5 \\ 7 & 5 \end{bmatrix}$.

B $\begin{bmatrix} 8 & -5 \\ -7 & 5 \end{bmatrix}$.

C $\begin{bmatrix} 4 & 7 \\ 1 & 3 \end{bmatrix}$.

D $\begin{bmatrix} -4 & 7 \\ 1 & -3 \end{bmatrix}$.

xii) Tashi Commercial Corporation sells varieties of products to its customers. In general, the total revenue it receives from selling x units of a product is given by $R(x)=20x-0.5x^2$. What is the marginal revenue generated from selling 10 units of the product?

A 10

B 15

C 20

D 25

xiii) The derivative of the function $y=sin^2(x^2)$ is

A $4x\ sin(x^2)$.

B $2x\ cos^2(x^2)$.

C $cos^2(x^2)$.

D $2x\ sin(2x^2)$.

xiv) Following are the ranks obtained by 6 students in two subjects, Statistics and Mathematics:

Statistics (*x*)	1	2	3	4	5	6
Mathematics (*y*)	2	4	1	5	3	9

In the above table, the Statistics and Mathematics marks have

A low degree negative correlation.

B high degree negative correlation.

C low degree positive correlation.

D moderate degree positive correlation.

xv) If $y = x^x$, then $\frac{dy}{dx}$ is

A $2x$.

B $x.x^{x-1}$.

C $x^x(1+\log x)$.

D $x^x(1-\log x)$.

SECTION B – [10 × 7 = 70 marks]

*Answer **any** 10 questions. All questions in this section have equal marks.*

Question 2

a) Calculate the semi inter-quartile range from the following distribution. **[3]**

Age in years	20	30	40	50	60	70	80
No.of persons	3	61	132	153	140	51	3

b) Find $\frac{dy}{dx}$ for the following functions. **[4]**

i) $y = x \log x - x$

ii) $x^3 + 8xy + y^3 = 64$

Question 3

a) For the parabola $y^2 = 18x$, find the coordinates of the focus, length of latus rectum and the equation of the directrix. **[3]**

b) A committee of 3 members is to be selected from amongst 5 boys and 6 girls.
In how many ways can this be done so as to include at least 1 boy? **[4]**

Question 4

a) The total revenue received from the sale of x units of a product is given by $R(x) = 20x + 5x^2 - 3x^3$. Find **[3]**

i. the average revenue,

ii. the marginal revenue and

iii. actual revenue from selling 10 units.

b) Compute $\int \frac{3x-2}{(x+1)^2(x+3)}\,dx$. **[4]**

Question 5

a) Using the properties of determinants, show that **[4]**

$$\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = xyz\,(x-y)(y-z)(z-x).$$

b) Nine counters numbered 2 to 10 are put in a bag. One counter is selected at random. What is the probability of getting a counter with **[3]**

i) an odd number,

ii) a multiple of 3 and

iii) a number 5?

Question 6

a) For $A=\begin{bmatrix} 2 & 2 & 2 \\ 2 & 1 & -3 \\ 1 & 0 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 3 & 3 \\ 3 & 0 & 5 \\ 6 & 9 & -1 \end{bmatrix}$, $C=\begin{bmatrix} 4 & 4 & 4 \\ 5 & -1 & 4 \\ 7 & 8 & -1 \end{bmatrix}$, **[3]**

Compute:

i. $3A-6B+9C$

ii. $7A-2B-C$

b) The correlation coefficient between the variables x and y is $r = 0.60$. If $\sigma_x = 1.50,\ \sigma_y = 2.00,\ \bar{X} = 10,\ \bar{Y} = 20,$ find the regression equations of y on x and x on y. **[4]**

Question 7

a) Dorji set up a poultry farm in his village. He borrowed Nu 100,000 from Bhutan Development Bank Ltd. on a condition to repay it with compound interest at 5 % p.a. at the annual installments of Nu 10,000 each. In how many years will his debt be liquidated? **[4]**

b) Determine the value of $\int (x \log x)\, dx$. **[3]**

Question 8

a) The given equation $4x^2 - 9y^2 - 8x - 32 = 0$ represents the equation of a conic.
Find its eccentricity and coordinates of the foci. **[4]**

b) How many numbers of 4 digits can be formed with the digits 1, 2, 3, 4 and 5 when

i. the digit is repeated?
ii. the digit is not repeated? **[3]**

Question 9

a) To save for a child's education, a family decides to invest Nu 3000 at the end of each six month period in a millennium scheme paying 8 % p.a. compounded annually. Find the amount of investment at the end of 18 years. **[3]**

b) Calculate the Karl Pearson's coefficient of correlation between the ages of husband and wife and interpret the result. **[4]**

Age of husband (*x*)	35	34	40	43	56	20	38
Age of wife (*y*)	32	30	31	32	53	20	33

Question 10

a) A function is defined by $f(x) = x^3 - 3x^2 - 9x + 7$. Determine its maximum and minimum values. **[4]**

b) A company sells its product at the rate of Nu 6 per unit. The variable costs are estimated to run 25 % of the total revenue received. If the fixed costs for the product is Nu 4500, find **[3]**

i. the total cost function,

ii. the profit function and

iii. the break-even point?

Question 11

a) The directrix of an ellipse is $3x + 4y = 1$ and focus is $(-2, 3)$. Find the equation of the ellipse if its eccentricity is $\frac{1}{\sqrt{2}}$. **[3]**

b) If $y = x^y$, prove that $x \frac{dy}{dx} = \frac{y^2}{1 - y \log x}$. **[4]**

Question 12

a) An analysis of daily wages of the workers of two organizations A and B yielded the following results:

	Organization	
	A	**B**
No. of workers	10	20
Average daily wages	Nu. 30	Nu. 15
Variance	25	100

Obtain the average daily wages. Which organization is more equitable with regard to wages? **[4]**

b) Integrate: $\int (3x^2+4x+5)^5 (3x+2)\,dx$. **[3]**

Question 13

a) A manufacturer can sell x items at a price of Nu$(250 - x)$ each. The cost of producing x items is Nu$(2x^2 - 50x + 12)$. **[4]**

i. Determine the number of items to be sold so that the manufacturer can make maximum profit.

ii. Find the maximum profit.

iii. Evaluate: $\int \frac{x^3+3x^2+2x+1}{x-1}\,dx$. **[3]**

Question 14

a) Show that the triangle with vertices $A(6,10,10)$, $B(1,0,-5)$ and $C(6,-10,0)$ is a right angled triangle. **[3]**

b) Solve the following system of equations using Cramer's rule. **[4]**

$$-4x + 2y - 9z = 2$$
$$3x + 4y + z = 5$$
$$x - 3y + 2z = 8$$

FORMULAE

CO-ORDINATE GEOMETRY

$$D=\sqrt{\left(x_2-x_2\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}$$

$$(x,y,z)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2},\frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$$

$a_1x+b_1y+c_1z=0$ and $a_2x+b_2y+c_2z=0$

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{z}{a_1b_2-a_2b_1}$$

$$\cos\theta=\pm\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^{\,2}+b_1^{\,2}+c_1^{\,2}}\sqrt{a_2^{\,2}+b_2^{\,2}+c_2^{\,2}}}$$

ALGEBRA

$a^2-b^2=(a+b)(a-b)$

$(a\pm b)^2=a^2\pm 2ab+b^2$

$In\,the\,quadratic\,equation\,ax^2+bx+c=0,\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$${}^{n}p_r=\frac{n!}{(n-r)!}$$

$${}^{n}C_r=\frac{n!}{r!(n-r)!}$$

$$C_{ij}=(-1)^{i+j}M_{ij}$$

$$AA^{-1}=A^{-1}A=I$$

$$A^{-1}=\frac{1}{\det A}\cdot adjA$$

$$x=\frac{D_x}{D},y=\frac{D_y}{D},z=\frac{D_z}{D}$$

COMMERCIAL MATHEMATICS

$$A=\frac{a}{i}(1+i)\left[(1+i)^n-1\right]$$

$$P=\frac{a}{i}\left[1-(1+i)^{-n}\right]$$

$$AC(x)=\frac{C(x)}{x},\ MC(x)=\frac{d}{dx}[C(x)]$$

$$C(x)=F+V(x)$$

$$R(x)=xG(x)....Output\times\ \text{Price}$$

$$P(x)=R(x)-C(x)$$

CALCULUS

$y = x^n$, $y'=nx^{n-1}$,

$$If\ y=u\pm v,then\ \frac{dy}{dx}=\frac{du}{dx}\pm\frac{dv}{dx}$$

$$If\ y=uv,then\ \frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$$

$$If\ y=\frac{u}{v},\ then\ \frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$$

$$\int uv\,dx=u\int v dx-\int\left(\frac{du}{dx}\int v dx\right)dx.$$

$$\int x^n .dx = \frac{x^{n+1}}{n+1} + c$$

$$\int (ax+b)^n = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

$$\int a^{nx}\ dx = \frac{a^{nx}}{n \log_e a} + c$$

DATA AND PROBABILITY

$$\overline{X} = \frac{\sum fx}{\sum f} \quad or \quad \bar{X} = \frac{\sum x}{n}$$

$$Median = L + \frac{i}{f}\left(\frac{N}{2} - c\right)$$

$$\sigma = \sqrt{\frac{\sum \left(x_i - \bar{x}\right)^2}{n}} \; or \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$\bar{x}_{12} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

$$\sigma_{12} = \sqrt{\frac{n_1 {\sigma_1}^2 + n_2 {\sigma_2}^2 + n_1 {d_1}^2 + n_2 {d_2}^2}{n_1 + n_2}}$$

$$Cov(X,Y) = \frac{1}{n} \sum \left(X - \bar{X}\right)\left(Y - \bar{Y}\right)$$

$$r = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \sum (y-\bar{y})^2}} = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

$$r = \frac{\sum \left(x - \bar{x}\right)\left(y - \bar{y}\right)}{n\, \sigma_x\, \sigma_y}$$

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}, \quad Correction\ factor = \frac{1}{12}\left(m^3 - m\right)$$

$$r = \pm \sqrt{b_{xy} \cdot b_{yx}}$$

$$b_{YX} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$b_{XY} = r\frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - \left(\sum y\right)^2}$$

$$Y - \bar{Y} = \frac{\mathrm{cov}(X,Y)}{{\sigma_x}^2}\left(X - \bar{X}\right) = r\frac{\sigma_y}{\sigma_x}\left(X - \bar{X}\right)$$

$$X - \bar{X} = \frac{\mathrm{cov}(X,Y)}{{\sigma_x}^2}\left(Y - \bar{Y}\right) = r\frac{\sigma_x}{\sigma_y}\left(Y - \bar{Y}\right)$$

$$\mathrm{b}_{xy} \times \mathrm{b}_{yx} = r\frac{\sigma_x}{\sigma_y} \times r\frac{\sigma_y}{\sigma_x}$$

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$$

$$x - \bar{x} = b_{xy}\left(y - \bar{y}\right)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A) + P\left(\bar{A}\right) = 1$$

$$P\left(B/A\right) = \frac{P(A \cap B)}{P(A)}$$

$$P\left(A/B\right) = \frac{P(A \cap B)}{P(B)}$$

Rough work

Rough work