SECTION A

(Answer **ALL** questions)

Direction: *For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than ONE alternative. If there are more than one choice circled,* ***NO*** *score will be awarded.*

Question 1 **[2×15 = 30]**

(i) If $y = \cos^{-1}\frac{1}{2} + 2\operatorname{cosec}^{-1}2$, then the value of y is equal to

A $\frac{\pi}{3}$.

B $\frac{\pi}{2}$.

C $\frac{2\pi}{3}$.

D $\frac{4\pi}{3}$.

(ii) A conic section formed by the intersection of right circular cone and a plane has different shapes depending on the ratio of locus of a point from the focus to the locus of a point from the directix. What determines the shape of a conic section?

A axes

B directrices

C latus rectum

D eccentricity

(iii) A school wants to raise fund to help the needy students by selling lottery. A lottery ticket should be of 5 digit number. How many tickets can be produced when the repetition of digit is allowed?

A 27216

B 30240

C 90000

D 100000

(iv) If the first order derivative of a function is $2x+1$, then the gradient of the function at $(-1,0)$ is

A -2.

B -1.

C 1.

D 2.

(v) Karma draws a card from a deck of 52 cards randomly and keeps it aside. What is the probability of getting an ace in second draw if the first card drawn was an ace?

A $\frac{3}{52}$

B $\frac{4}{52}$

C $\frac{3}{51}$

D $\frac{4}{51}$

(vi) Dorji has15 friends, out of which 7 are boys and 8 are girls. In how many ways can he invite 10 friends for his birthday party such that 4 of them are girls?

A 210

B 490

C 1365

D 3003

(vii) If $\int_0^a 3x^2 dx = 27$, then the value of "a" is

A 3.

B -3.

C 9.

D -9.

(viii) The angle between the planes $2x - y + z = 7$ and $x + y + 2z - 11 = 0$ is

A 30^o.

B 45^o.

C 60^o.

D 90^o.

(ix) Find the standard deviation of the first five prime numbers.

A 3.2

B 6

C 10.3

D 11.2

(x) Identify the quadrant in which the complex number $z = a + bi$ lies, if $a > 0$ and $b < 0$.

A first quadrant

B second quadrant

C third quadrant

D fourth quadrant

(xi) For the system of equations $x + y = 4$ and $2x + 2y = 2$, which of the following is true?

A It has unique solution.

B It has three solutions.

C It has infinite solutions.

D No solution.

(xii) $\int \frac{f'(x)}{f(x)}dx = \log[f(x)] + C$. Using the given information, find the value of $\int \frac{x+2}{x^2+4x}dx$.

A $\frac{1}{2}\log(x^2+4x)+C$

B $\log(x^2+4x)+C$

C $\frac{1}{2}\log(x+2)+C$

D $\log(x+2)+C$

(xiii) If $f'(x) = cosx + sinx$ and $f(0) = 1$, then $f(\frac{\pi}{2})$ is

A 0.

B 1.

C 2.

D 3.

(xiv) If the equation $x^2 - y^2 = 0$ represents a pair of straight lines, then the two lines and their point of intersection is

A $x + y = 0,\ xy = 0,\ (0,0)$.

B $x + y = 0,\ x - y = 0,\ (0,0)$.

C $x + y = 0,\ x - y = 0,\ (1,0)$.

D $x + y = 1,\ x - y = 0,\ \left(\frac{1}{2},\frac{1}{2}\right)$.

(xv) If $A(2,2,3)$ is a point in 3-D, then what will be the distance of A from the origin?

A $2\sqrt{2}$ unit

B $\sqrt{13}$ unit

C $\sqrt{17}$ unit

D 17 unit

SECTION B [70 Marks]

*Answer any **10** questions. All questions in this section have equal marks.*

Question 2

(a) Without expanding the determinants, state and use the properties to show that **[4]**

$$\begin{vmatrix} \frac{1}{x} & x^2 & yz \\ \frac{1}{y} & y^2 & zx \\ \frac{1}{z} & z^2 & xy \end{vmatrix} = 0.$$

(b) What is the derivative of $y = x^x$? **[3]**

Question 3

(a) Show that $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$. **[4]**

(b) You are given 5 vowels and 10 consonants to form different words. How many different words, each containing 3 vowels and 4 consonants can be formed? **[3]**

Question 4

(a) State the degree and order of the differential equation $\frac{dy}{dx} = \frac{2y}{x} - x^2$ and solve it. **[4]**

(b) Determine whether the two points $(0,0,0)$ and $(2,-4,3)$ lie on the same side or opposite sides of the plane $x+3y-5z+7=0$. **[3]**

Question 5

(a) Determine the value of $\left(1+\omega-\omega^2\right)\left(1-\omega+\omega^2\right)$. **[2]**

(b) The system of linear equations are as given below: **[5]**

$$x + y = 5$$
$$z + y = 7$$
$$z + x = 6$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 7 \\ 6 \end{bmatrix}$$

Using the matrix A and matrix B, find:

i. $|A|$

ii. Adj(A)

iii. A^{-1}

iv. The values of x, y and z

Question 6

(a) Solve the equation: $\cos\left(\sin^{-1} x\right) = \frac{1}{3}$. **[3]**

(b) The equation $8x^2+8xy+2y^2+26x+13y+15=0$ represents a pair of straight lines.

i. Find the equation of the two lines.

ii. Compare the slopes of the two lines and explain your observation. **[4]**

Question 7

(a) Sketch the curve $y^2 = 12x$ and determine the area and volume as directed below:

i. Area of the region bounded by the curve and its latus rectum

ii. Volume generated when the same region is rotated 360° about x- axis **[4]**

(b) The equation $4x^2 + 5y^2 = 100$ represents one of the conic sections.

i. Convert the equation to standard form and identify the conic section.

ii. Determine eccentricity.

iii. Find the equation of directrices. **[3]**

Question 8

(a) Evaluate: $\int \frac{x+1}{x^2+4x-5}dx$. **[4]**

(b) Write the equation of parabola whose vertex is at $(0,0)$, passing through $(-3,7)$ and axis along the x- axis. **[3]**

Question 9

(a) What should be the dimensions of a rectangular kitchen garden with an area of $25\ m^2$, so that the materials used to fence is minimum? **[4]**

(b) For what value of 'x' will the line through $(4,1,2)$ and $(5,x,0)$ be parallel to the line through $(2,1,1)$ and $(3,3,-1)$? **[3]**

Question 10

(a) A mathematical problem is given to four students whose chances of solving a problem are $\frac{1}{5}, \frac{1}{4}, \frac{1}{3}$ and $\frac{1}{2}$ respectively. What is the probability that the problem will be solved? **[4]**

(b) Show that the following points lie in the same plane:

$(-6,3,2)$, $(3,-2,4)$, $(5,7,3)$ and $(-13,17,-1)$ **[3]**

Question 11

(a) Find the centre and eccentricity of the hyperbola
$9x^2 - 16y^2 - 18x - 64y - 199 = 0$. **[4]**

(b) Evaluate: $\int x^2 e^x dx$. **[3]**

Question 12

(a) Using De Moivre's theorem, find the values of $(1+i)^{\frac{1}{2}}$. **[4]**

(b) Find the second order derivative of $y = 2\sin x + 3\cos x$. Use it to verify the equation $y + \frac{d^2y}{dx^2} = 0$. **[3]**

Question 13

(a) Differentiate the function $x^2 + y^2 = \log(xy)$. **[3]**

(b) $3x+12y=9$ and $9x+3y=46$ are regression equations of y on x and x on y respectively.

i. Is the above statement true? Justify your answer.

ii. Predict the value of y when $x=10$. **[4]**

Question 14

(a) The marks obtained by 8 students in Mathematics and English are as follows: **[4]**

Marks in Mathematics	55	60	70	65	85	35	72	63
Marks in English	50	45	75	54	60	62	51	43

i. Compute the rank in the two subjects and coefficient of correlation.

ii. To what extent the knowledge of the students in the two subjects are related?

(b) Illustrate $|Z| = 3$ in complex plane and explain your answer. **[3]**

FORMULAE

Trigonometry

$$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

$$\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right)$$

$$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left(\frac{x \pm y}{1 \mp xy}\right), xy < 1$$

$$2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2} = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}$$

$$\operatorname{cosec}^{-1}x = \sin^{-1}\frac{1}{x}$$

$$\sec^{-1} x = \cos^{-1}\frac{1}{x}$$

$$\cot^{-1} x = \tan^{-1}\frac{1}{x}$$

Complex Numbers

$$r = \sqrt{a^2 + b^2}$$

$$\tan\theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

If $z = r(\cos\theta + i\sin\theta)$ then

$$z^n = r^n(\cos n\theta + i\sin n\theta)$$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{2k\pi+\theta}{n} + i\sin\frac{2k\pi+\theta}{n}\right),$$

$$k = 0, 1, 2, 3, \ldots n-1$$

Co-ordinate Geometry

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$(x, y, z) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$$

$a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

Angle between two planes,

$$\cos\theta = \pm\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

distance of a point from a plane $= \pm\dfrac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$

$$(x, y) = \left(\frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \frac{m_1y_1 + m_2y_2}{m_1 + m_2}\right)$$

Angle between the lines : $ax^2 + 2hxy + by^2 = 0$,

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$$

equation of bisector, $\dfrac{x^2 - y^2}{a-b} = \dfrac{xy}{h}$

points of intersection, $\left(\dfrac{hf - bg}{ab - h^2}, \dfrac{gh - af}{ab - h^2}\right)$

Algebra

$$a^2 - b^2 = (a+b)(a-b)$$
$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$
$$\text{In the quadratic equation } ax^2 + bx + c = 0,\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$^n p_r = \frac{n!}{(n-r)!}$$

$$^nC_r = \frac{n!}{r!(n-r)!}$$

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$AA^{-1} = A^{-1}A = I$$

$$A^{-1} = \frac{1}{\det A} \cdot adjA$$

$$x = \frac{D_x}{D}, y = \frac{D_y}{D}, z = \frac{Dz}{D}$$

$$1 + 2 + 3 + \ldots.. + (n-1) = \frac{1}{2} n(n-1)$$
$$1^2 + 2^2 + 3^2 + \ldots\ldots + (n-1)^2 = \frac{1}{6} n(n-1)(2n-1)$$
$$1^3 + 2^3 + 3^3 + \ldots\ldots + (n-1)^3 = \left\{\frac{n(n-1)}{2}\right\}^2$$

Calculus

$y = x^n$, $y' = nx^{n-1}$,

$y = cf(x)$, $y' = cf'(x)$,

$$\text{If } y = u \pm v, \text{then } \frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$\text{If } y = uv, \text{then } \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\text{If } y = \frac{u}{v}, \text{ then } \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int uv\,dx = u\int v dx - \int\left(\frac{du}{dx}\int v dx\right)dx.$$

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[\sum_{r=0}^{n-1} f(a+rh)\right]$$

$$\frac{dy}{dx} + py = Q, I.F = ye^{\int pdx},$$
$$\text{general solution, } y.IF = \int (Q.IF)dx + c$$

$$V = \pi\int_a^b y^2 dx \quad A = \int_a^b y dx$$

$$\text{Volume of Cone} = \frac{1}{3}\pi r^2 h$$
$$\text{Volume of Sphere} = \frac{4}{3}\pi r^3$$
$$\text{Volume of Cylinder} = \pi r^2 h$$
$$S.\text{Area of Cone} = \pi rl + \pi r^2$$
$$S.\text{Area of Sphere} = 4\pi r^2$$
$$S.\text{Area of Cylinder} = 2\pi rh + 2\pi r^2$$

Data and Probability

$$\overline{X} = \frac{\sum fx}{\sum f} \quad or \quad \overline{X} = \frac{\sum x}{n}$$

$$Median = L + \frac{i}{f}\left(\frac{N}{2} - c\right)$$

$$\sigma = \sqrt{\frac{\sum\left(x_i - \overline{x}\right)^2}{n}} \; or \; \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$\overline{X} = \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2}$$

$$Mean\,Deviation = \frac{\sum f\left(x - \overline{x}\right)}{\sum f}$$

$$\sigma_{12} = \sqrt{\frac{n_1\sigma_1^{\ 2} + n_2\sigma_2^{\ 2} + n_1 d_1^{\ 2} + n_2 d_2^{\ 2}}{n_1 + n_2}}$$

$$\text{Cov}(\text{X}, \text{Y}) = \frac{1}{\text{n}}\sum\left(\text{X} - \overline{\text{X}}\right)\left(\text{Y} - \overline{\text{Y}}\right)$$

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2 \sum(y - \bar{y})^2}} = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{n\sigma_x\,\sigma_y}$$

$$r = 1 - \frac{6\sum d^2}{n(n^2 - 1)}, \; Correction\ factor = \frac{1}{12}\left(m^3 - m\right)$$

$$r = \pm\sqrt{b_{xy}.b_{yx}}$$

$$b_{YX} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$b_{XY} = r\frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - \left(\sum y\right)^2}$$

$$Y - \bar{Y} = \frac{\text{cov}\left(X, Y\right)}{\sigma_x^{\ 2}}\left(X - \bar{X}\right) \; = \; r\frac{\sigma_y}{\sigma_x}\left(X - \bar{X}\right)$$

$$X - \bar{X} = \frac{\text{cov}\left(X, Y\right)}{\sigma_x^{\ 2}}\left(Y - \bar{Y}\right) \; = \; r\frac{\sigma_x}{\sigma_y}\left(Y - \bar{Y}\right)$$

$$\text{b}_{xy} \times \text{b}_{yx} = r\frac{\sigma_x}{\sigma_y} \times \; r\frac{\sigma_y}{\sigma_x}$$

$$\sum y = na + b\sum x$$
$$\sum xy = a\sum x + b\sum x^2$$

$$y - \overline{y} = b_{yx}\left(x - \overline{x}\right)$$
$$x - \overline{x} = b_{xy}\left(y - \overline{y}\right)$$

$$P\left(A \cup B\right) = P\left(A\right) + P\left(B\right) - P\left(A \cap B\right)$$

$$P\left(A\right) + P\left(\overline{A}\right) = 1$$

$$P\left(B/A\right) = \frac{P\left(A \cap B\right)}{P\left(A\right)}$$

$$P\left(A/B\right) = \frac{P\left(A \cap B\right)}{P\left(B\right)}$$

Rough work

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