SECTION A [30 MARKS]

ANSWER **ALL** QUESTIONS

For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than ONE alternative. If there are more than one choice circled, NO score will be awarded.

Question 1 **[30]**

i. What is the present value of perpetual annuity of Nu 1000 at the interest rate of 5% per annum?

A Nu 200
B Nu 2000
C Nu 20000
D Nu 200000

ii. What value of 'r' will make the relation ${}^{11}C_{2r} = {}^{11}C_{2+r}$ true, if $2r \neq r+2$?

A 3
B 9
C 10
D 12

iii. If $\frac{dy}{dx}$ of the function is $2x^2+5$, then the gradient of the function at $(-1,0)$ will be

A - 4.
B 4.
C 5.
D 7.

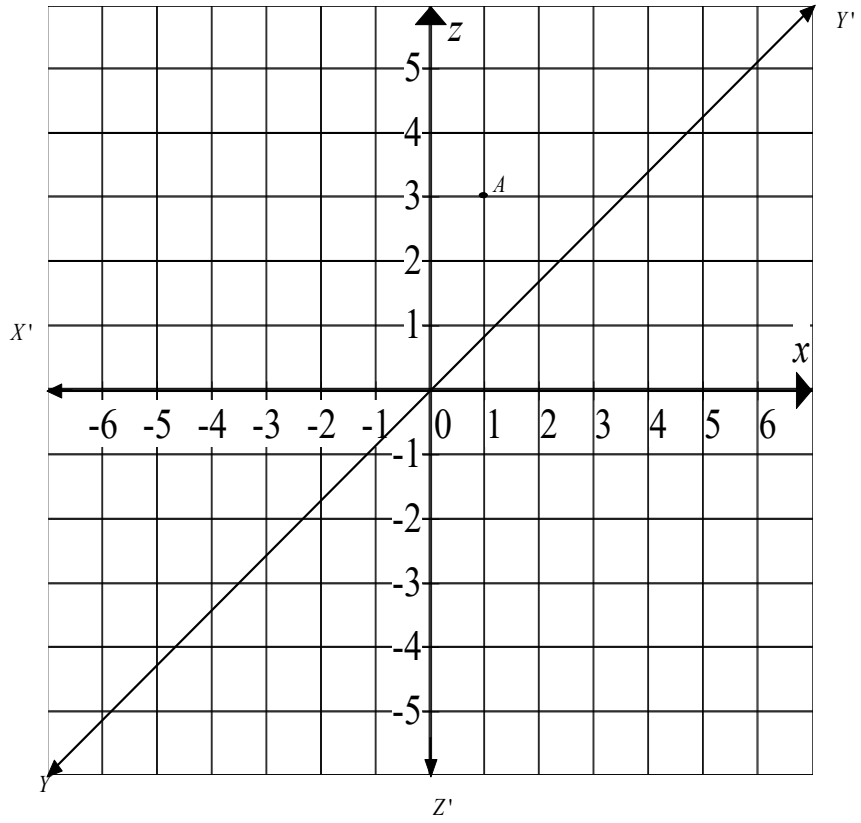
iv. Find the probability that the first die shows 6, when two unbiased dice are thrown.

A $\frac{5}{6}$
B $\frac{5}{36}$
C $\frac{1}{6}$
D $\frac{1}{36}$

v. The total revenue received from the sale of 'P' units of oranges is given by $R(P) = 10P + 2P^2 + 100$. Find the revenue on an average by selling 50 units.

A Nu 112
B Nu 210
C Nu 5500
D Nu 5600

vi. From the diagram below, determine the distance of the point A $(x, 1, 4)$ from the origin.



A $\sqrt{17}$
B $\sqrt{20}$
C $\sqrt{21}$
D $\sqrt{25}$

vii. If three brothers x, y and z have to always sit together, in how many ways can they arrange themselves so that the younger brother 'x' has to always sit in the middle?

A 1
B 2
C 3
D 6

viii. Evaluate $\int (2x+1)^3 . dx$

A $\frac{(2x+1)^4}{4} + C$

B $\frac{(2x+1)^4}{8} + C$

C $(6x+3)^2 + C$

D $3(2x+1)^2 + C$

ix. Find the value of x, if the line through $A(4,1,2)$ and $B(5,x,0)$ is parallel to the line through $C(2,1,1)$ and $D(3,3,-1)$.

A 1

B 2

C 3

D 4

x. Find the second order derivative of the function $y = (1-x)(x+1)$.

A 0

B $-2x$

C $+2$

D -2

xi. If the two lines of regression when plotted on a graph coincide with the slope greater than zero, there will be

I perfect

II high degree

III low degree

IV positive correlation

V negative correlation

A I and IV.

B II and IV.

C I and V.

D III and V.

xii. For what value of 'a', the matrix $X = \begin{bmatrix} a & 1 & 3 \\ 2 & 2 & 6 \\ 2 & -3 & 1 \end{bmatrix}$ will be singular?

A 0

B 1

C 2

D 3

xiii. What amount has Mr. Dawa to deposit in a bank at the end of each year at 5% interest rate, so that his accumulation at the end of 15 years will be Nu 107920?

A Nu 4500
B Nu 5000
C Nu 10000
D Nu 10500

xiv. What is the integral of $\frac{6x}{x^2}$ with respect to x ?

A $\log x+c$
B $\log x^2+c$
C $6\log x+c$
D $-6x^{-2}+c$

xv. A die is thrown and the outcome is an odd number. What is the probability of getting a prime number?

A 1
B $\frac{1}{2}$
C $\frac{1}{3}$
D $\frac{2}{3}$

SECTION B [70 MARKS]

ATTEMPT ANY **10** QUESTIONS

Question 2

a) The marks obtained by Class XII Students in Business Mathematics and in Economics are as follows: [4]

Marks in Business Mathematics:	35	23	47	17	10	43	9	6	28
Marks in Economics:	30	33	45	23	8	49	12	4	31

Compute their ranks in the two subjects and coefficient of correlation of ranks.

b) For $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, find $A^2 - 4A + 7I$. [3]

Question 3

a) Determine the maximum profit that a company can generate if the profit function is given by $P(x)=52x-x^2-100$. [4]

b) Using determinants, find the area of the triangle with the vertices $A(-3,5), B(3,-6)$ and $C(7,2)$. [3]

Question 4

a) Find the anti-derivative of $\frac{2x-5}{\sqrt{x^2-5x+3}}$ with respect to x. [3]

b) A company wants to launch a new product by investing Nu 35000 as the fixed cost and Nu 500 per unit as the variable cost of production. The revenue function for the sale of x units is given by $5000x - 100x^2$. Find the value of x at the point where there will be no loss or no gain. [4]

Question 5

a) Find the coordinates of point $A(x, y, z)$ in between the points $B(1,3,7)$ and $C(6,3,2)$ in the ratio $2:3$. [3]

b) Find $\frac{dy}{dx}$ in $4x^2 + y^2 - xy + 2y - 2 = 0$ [4]

Question 6

a) The marks obtained by 5 students in Mathematics and Accountancy tests are given below. [4]

Mathematics	20	13	18	21	11
Accountancy	17	12	23	25	14

Calculate Karl Pearson's correlation coefficient and interpret the result.

b) In how many ways can a student choose 10 questions out of 13 questions if 5 questions are compulsory? [3]

Question 7

a) Write the system of equation $x+y=5$, $z+y=7$, $z+x=6$ in determinant form and find

i. determinant. [2]

ii. the values of x, y and z. [2]

b) The cost function is given by $3x^2 - 2x+5$, find the

i. average cost. [1]

ii. marginal cost. [1]

iii. marginal cost when $x = 4$ [1]

Question 8

a) Mr. Pema and Mrs. Dema appear for an interview for the same post. The probability of selecting Mr. Pema is $\frac{1}{7}$ and that of Mrs. Dema is $\frac{1}{5}$. What is the probability that;

[1½]

i. both of them will be selected?

ii. only one of them will be selected? [1½]

b) Tobgay buys a mobile phone paying Nu 4000 in cash and promising to pay Nu 200 at the end of every month for the next 4 years. If money is worth 12% p.a, converted monthly, what will be the cash price of the mobile phone? [4]

Question 9

a) Evaluate $\int \frac{x+7}{(x-2)(x+4)} dx$. [4]

b) If Nu 1000 is paid at the rate of 8% per annum compounded annually, find the number of years for the amount to exceed Nu 20000. [3]

Question 10

a) Verify that $\frac{dy}{dx} = \frac{2}{2y-1}$,if $y = \sqrt{2x + \sqrt{2x + \sqrt{2x + \cdots \text{to } \infty}}}$ [3]

b) Express the $|A| = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & \gamma & \gamma^2 \end{vmatrix}$ into the factors $(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$ using the properties of determinant. [4]

Question 11

a) Equation of two regression lines are $4x - 5y = -33$ and $20x - 9y = 107$.

From the above equation of lines, find

i. Coefficient b_{yx} and Coefficient b_{xy} [2]

ii. the mean value of x and y. [2]

b) Evaluate $\int (2x^3 + 4)x^2 . dx$ [3]

Question 12

a) If $f(x) = \frac{\sqrt{x-1}-\sqrt{x+1}}{\sqrt{x-1}+\sqrt{x+1}}$, Find $f'(2)$. [3]

b) At the beginning of each quarter, Nu 2000 is deposited in the savings account which pays an interest of 10% p.a compounded quarterly. Find the balance in the account after 5 years. [4]

Question 13

a) Find the angle θ between AB and CD for given coordinates: [3]

A(-3,2,4), B(2,5,-2), C(1,-2,2) and D(4,2,3).

b) Integrate the following functions.

i. $\int \frac{(x^2-9)}{(x-3)} . dx$ [2]

ii. $\int (x-1)(x^2+x+1).\,dx$ [2]

Question 14

a) If $\cos\alpha, \cos\beta$ and $\cos\gamma$ are the direction cosines of the line with $\cos\alpha = \frac{14}{15}$, $\cos\beta = \frac{1}{3}$. Determine $\cos\gamma$. [3]

a) Find the inverse of the matrix $A=\begin{bmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ -6 & 4 & 7 \end{bmatrix}$. [4]

MATHEMATICAL FORMULAE

Co-ordinate Geometry

$$D=\sqrt{\left(x_2-x_2\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}$$

$$(x,y,z)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2},\frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$$

$$a_1x+b_1y+c_1z=0 \text{ and } a_2x+b_2y+c_2z=0$$

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{z}{a_1b_2-a_2b_1}$$

$$\cos\theta=\pm\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^{\,2}+b_1^{\,2}+c_1^{\,2}}\sqrt{a_2^{\,2}+b_2^{\,2}+c_2^{\,2}}}$$

Commercial Mathematics

$$A=\frac{a}{i}(1+i)\left[(1+i)^n-1\right]$$

$$A=\frac{a}{i}\left[(1+i)^n-1\right]$$

$$P=\frac{a}{i}\left[1-(1+i)^{-n}\right]$$

$$P=\frac{a}{i}(1+i)\left[1-(1+i)^{-n}\right]$$

$$A(x)=\frac{C(x)}{x},\ M(x)=\frac{d}{dx}(C(x))$$

$$C(x)=F+V(x)$$

$$R(x)=xG(x)$$

$$P(x)=R(x)-C(x)$$

$$MC=\frac{d}{dx}(C(x))$$

Algebra

$$a^2-b^2=(a+b)(a-b)$$

$$(a\pm b)^2=a^2\pm 2ab+b^2$$

$$\text{In the quadratic equation } ax^2+bx+c=0,$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$${}^nP_r=\frac{n!}{(n-r)!}$$

$${}^nC_r=\frac{n!}{r!(n-r)!}$$

Calculus

$y = x^n,\ y' = nx^{n-1},$

$$If\ y=u\pm v,\ then\ \frac{dy}{dx}=\frac{du}{dx}\pm\frac{dv}{dx}$$

$$If\ y=uv,\ then\ \frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$$

$$If\ y=\frac{u}{v},\ then\ \frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$A A^{-1} = A^{-1} A = I$$

$$A^{-1} = \frac{1}{\det A} \cdot adjA$$

$$x = \frac{D_x}{D}, y = \frac{D_y}{D}, z = \frac{Dz}{D}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int uv \, dx = u \int v dx - \int \left(\frac{du}{dx} \int v dx \right) dx.$$

Data and Probability

$$\bar{x} = \frac{\sum fx}{\sum f} \quad or \quad \bar{x} = \frac{\sum x}{n}$$

$$Median = L + \frac{i}{f}\left(\frac{N}{2} - c \right)$$

$$\sigma = \sqrt{\frac{\sum \left(x_i - \bar{x} \right)^2}{n}} \, or \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n} \right)^2}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

$$\overline{X_{12}} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

$$\sigma_{12} = \sqrt{\frac{n_1 {\sigma_1}^2 + n_2 {\sigma_2}^2 + n_1 {d_1}^2 + n_2 {d_2}^2}{n_1 + n_2}}$$

$$\text{Cov}(\text{X}, \text{Y}) = \frac{1}{\text{n}} \sum \left(\text{X} - \overline{\text{X}} \right)\left(\text{Y} - \overline{\text{Y}} \right)$$

$$r = \frac{\sum \left(x - \bar{x} \right)\left(y - \bar{y} \right)}{\sqrt{\sum \left(x - \bar{x} \right)^2 \sum \left(y - \bar{y} \right)^2}} = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - \left(\sum x \right)^2} \sqrt{n \sum y^2 - \left(\sum y \right)^2}}$$

$$r = \frac{\sum \left(x - \bar{x} \right)\left(y - \bar{y} \right)}{n \sigma_x \sigma_y}$$

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}, \quad Correction\ factor = \frac{1}{12}\left(m^3 - m\right)$$

$$r = \pm\sqrt{b_{xy.}b_{yx}}$$

$$b_{YX} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$b_{XY} = r\frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x\sum y}{n\sum y^2 - \left(\sum y\right)^2}$$

$$Y - \bar{Y} = \frac{\mathrm{cov}(X,Y)}{\sigma_x^{\,2}}\left(X - \bar{X}\right) = r\frac{\sigma_y}{\sigma_x}\left(X - \bar{X}\right)$$

$$X - \bar{X} = \frac{\mathrm{cov}(X,Y)}{\sigma_x^{\,2}}\left(Y - \bar{Y}\right) = r\frac{\sigma_x}{\sigma_y}\left(Y - \bar{Y}\right)$$

$$\mathrm{b}_{xy} \times \mathrm{b}_{yx} = r\frac{\sigma_x}{\sigma_y} \times r\frac{\sigma_y}{\sigma_x}$$

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$$

$$x - \bar{x} = b_{xy}\left(y - \bar{y}\right)$$

$$P\left(A \cup B\right) = P\left(A\right) + P\left(B\right) - P\left(A \cap B\right)$$

$$P\left(A\right) + P\left(\bar{A}\right) = 1$$

$$P\left(B/A\right) = \frac{P\left(A \cap B\right)}{P\left(A\right)}$$

$$P\left(A/B\right) = \frac{P\left(A \cap B\right)}{P\left(B\right)}$$

ROUGH WORK

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