SECTION A [30 MARKS]

ANSWER **ALL** QUESTION

For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than ONE alternative. If there are more than one choice circled, NO score will be awarded.

Question 1 **[30]**

i. What is the interpretation of correlation coefficient if the value of 'r' is 0.5 to 0.699?

A high degree
B moderate degree
C low degree
D no correlation

ii. Find 'n' if ${}^4P_2 = n.{}^4C_2$.

A 0
B 1
C 2
D 3

iii. What is the value of $\cos^{-1}\left(-\frac{1}{2}\right)$?

A $\frac{2\pi}{3}$
B $-\frac{\pi}{2}$
C $-\frac{\pi}{3}$
D $\frac{\pi}{3}$

iv. The coordinates of the point which is two-fifths of the way from $A(3,4,5)$ to $B(-2,-1,0)$ is

A $(1,2,3)$.
B $(0,1,2)$.
C $\left(\frac{11}{5},\frac{18}{5},5\right)$.
D $\left(-\frac{4}{5},1,2\right)$.

v. Modify $\left(1+\sqrt{3}i\right)$ in trigonometric form of complex number.

A $\left(\cos\frac{\pi}{3}+\sin\frac{\pi}{3}i\right)$

B $2\left(\cos\frac{\pi}{3}+\sin\frac{\pi}{3}i\right)$

C $\left(2+60i\right)$

D $2\left(2+60i\right)$

vi. If $A=\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2=\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

A $\alpha=a^2+b^2, \beta=2ab$.

B $\alpha=a^2-b^2, \beta=2ab$

C $\alpha=a^2+b^2, \beta=ab$.

D $\alpha=a^2+b^2, \beta=a+b$.

vii. If $f\left(x\right)=3x^2-4x$, solve for 'a' given $f'(a) = 2$.

A 4
B 3
C 2
D 1

viii. The eccentricity of the curve $5x^2-4y^2=20$ is

A $\frac{2}{3}$.

B $\frac{1}{2}$.

C $\frac{3}{2}$.

D $\frac{1}{4}$.

ix. Find the probability that the sum of the two numbers obtained is 5 or 7, when a die is thrown twice.

A $\frac{1}{54}$

B $\frac{5}{18}$

C $\frac{1}{18}$

D $\frac{2}{3}$

x. The simplified expression of the determinant $\begin{vmatrix} 3x+y & 2x & x \\ 4x+3y & 3x & 3x \\ 5x+6y & 4x & 6x \end{vmatrix}$ is

A x.

B x^2+1.

C x^3.

D x^4+2.

xi. The integrating factor of the differential equation $x\frac{dy}{dx}+y=x^3$ is

A $\frac{1}{x}$.

B x

C $\frac{1}{x^2}$.

D x^2

xii. $\int_0^{\pi}\frac{dx}{1+\sin x}$ is

A 0.

B 1.

C 2.

D ∞.

xiii. For what value of λ does the equation $3x^2+2\lambda xy-3y^2-40x+30y-75=0$ represent two lines?

A 4

B -4

C 16

D -16

xiv. $\int \frac{3e^{2x}+3e^{4x}}{e^{x}+e^{-x}}dx$ is

A $\frac{e^{x}+c}{3}$.

B $\frac{e^{3x}}{3}+c$.

C $e^{x}+c$.

D $e^{3x}+c$.

xv. The distance between $x+2y-z+3=0$ and $3x+6y-3z+7=0$ is

A $\frac{\sqrt{6}}{9}$.

B $\frac{9}{\sqrt{6}}$.

C $\sqrt{54}$.

D $\sqrt{6}$.

SECTION B [70 MARKS]

ATTEMPT ANY **10** QUESTIONS

Question 2

a) If $\sin^{-1}\left(\sqrt{1-x^{2}}\right)+\cos^{-1}y+\tan^{-1}\frac{\sqrt{1-z^{2}}}{z}=180^{0}$, prove that $x^{2}+y^{2}+z^{2}=1-2xyz.$ [3]

b) Dawa found the correlation coefficient between x and y as 0.75 from the equation of two regression lines $3x+4y+8=0$ and $4x+3y+7=0$. Compare your finding with that of Dawa. [4]

Question 3

a) Solve the system of linear equations: $3x-2y+2z=5,\ -x+2y+z=6,\ x+2y-z=2$ [5]

using inverse of the given matrix $\begin{bmatrix} 3 & -2 & 2 \\ -1 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$.

b) Find the area between the curve $y=-x^2+4x$ and the x-axis from $x=0$ to $x=5$. [2]

Question 4

a) The following table gives the point scored by seven contestants out of 10 points. Interpret the points given by judge 'X' and 'Y'. Compute the Karl Pearson's Coefficient of Correlation. [3]

Judge	Contestants						
	A	**B**	**C**	**D**	**E**	**F**	**G**
X	5	8	3	7	3	9	7
Y	4	7	3	9	7	5	7

b) Solve for x: $\tan^{-1}2x+\tan^{-1}3x=\tan^{-1}\left(2\sin\frac{\pi}{4}\cos\frac{\pi}{4}\right)$. [4]

Question 5

a) In how many ways can a group of 4 members be selected from 5 boys and 5 girls including at least 2 girls? [2]

b) Find the equation of the plane perpendicular to each of the planes $x-2y-8z=0$ and $2x+5y-z=0$ and passing through the point $(-1,3,2)$. Also find the angle between the above given planes. [5]

Question 6

a) The line through (4,1,2) and (5, x,0) is parallel to the line through (2,1,1) and $(3,3,-1)$. Find the value of x. [2]

b) A closed box with a square base of side *x* centimeters with height *h* centimeters is to be constructed so as to contain 1000cm^3 of grains. Prove that the expenses of painting the inside of the box would be least if $h=x$. [5]

Question 7

a) Evaluate integrals as limit of sums: $\int_0^3 \left(x^2+x\right)dx$ [3]

b) Find the equations of the lines represented by the equation [4]

$4x^2+11xy+6y^2-x+3y-3=0$.

Question 8

a) What is the equation of parabola whose focus is (1,2) and equation of directrix is $2x+3y=1$? [3]

b) Differentiate x^{x^x}. [4]

Question 9

a) Illustrate the set of points satisfying $\arg(z+a)=\frac{\pi}{4}$ in the Argand's plane. [3]

Explain your answer.

b) The chances of three students passing the Preliminary Examination in RCSE are 50%, 60% and 70%. What is the probability that at least one of them passes? Express your answer in percentage. [4]

Question 10

a) Integrate $\int x(\log x)^2 dx$. [3]

b) Find the value of a determinant using the properties of determinant: $\begin{vmatrix} p^2 & q^2 & r^2 \\ qr & rp & pq \\ p & q & r \end{vmatrix}$ [4]

Question 11

a) Find the volume of a solid generated by revolving the area bounded by the parabola $y^2=8x$ and its latus rectum about the latus rectum. [3]

b) Using De Moivre's theorem, find the least value of n for which $\left(\frac{1}{2}+\frac{1}{2}i\right)^{n}$ is purely imaginary. [4]

Question 12

a) Find the solution of differential equation: $\frac{dy}{dx}=\frac{\sqrt{x^2+y^2}+y}{x}$ [4]

b) The entrance aptitude test score of 10 architect students and their aggregate marks at the end of their course in a college are given below. [3]

Student	A	B	C	D	E	F	G	H	I	J
Aptitude Score	2	5	0	4	3	1	6	8	7	9
Aggregate marks	8	16	8	9	5	4	3	17	8	12

Calculate the coefficient of rank correlation. Comment on your result.

Question 13

a) Find the square root of the complex number: $32i$. [3]

b) Find its area if $(6,10,10),(1,0,-5)$ and $(6,-10,0)$ represent vertices of a right angled triangle. [4]

Question 14

a) Find $\frac{dy}{dx}$ if $\sqrt{\frac{y}{x}}+\sqrt{\frac{x}{y}}=6$. [3]

b) Find the centre, length of axes and coordinates of vertices of the ellipse: $16x^2+25y^2-32x-200y+16=0$. [4]

MATHEMATICS FORMULAE

Trigonometry

$$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

$$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left(\frac{x \pm y}{1 \mp xy}\right), xy < 1$$

$$2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2} = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}$$

$$\operatorname{cosec}^{-1}x = \sin^{-1}\frac{1}{x}$$

$$\sec^{-1} x = \cos^{-1}\frac{1}{x}$$

$$\cot^{-1} x = \tan^{-1}\frac{1}{x}$$

Co-ordinate Geometry

$$D = \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$(x, y, z) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$$

$a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

Angle between two planes,

$$\cos\theta = \pm\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

distance of a point from a plane

$$= \pm\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$$

$$(x, y) = \left(\frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \frac{m_1y_1 + m_2y_2}{m_1 + m_2}\right)$$

Angle between the lines $ax^2 + 2hxy + by^2 = 0$,

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

Complex Numbers

$$r = \sqrt{a^2 + b^2}$$

$$\tan\theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

If $z = r(\cos\theta + i\sin\theta)$ then

$$z^n = r^n(\cos n\theta + i\sin n\theta)$$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{2k\pi + \theta}{n} + i\sin\frac{2k\pi + \theta}{n}\right),$$

$$k = 0, 1, 2, 3, \ldots n-1$$

Algebra

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

In the quadratic equation $ax^2 + bx + c = 0,\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$^nP_r = \frac{n!}{(n-r)!}$$

$$^nC_r = \frac{n!}{r!(n-r)!} \qquad C_{ij} = (-1)^{i+j} M_{ij}$$

$$A^{-1} = \frac{1}{\det A} \cdot adjA$$

$$1 + 2 + 3 + ... + (n-1) = \frac{1}{2} n(n-1)$$

$$1^2 + 2^2 + 3^2 + ... + (n-1)^2 = \frac{1}{6} n(n-1)(2n-1)$$

$$1^3 + 2^3 + 3^3 + ... + (n-1)^3 = \left\{ \frac{n(n-1)}{2} \right\}^2$$

$$A\,A^{-1} = A^{-1}A = I$$

$$V = \pi \int_a^b y^2 dx \qquad A = \int_a^b y dx$$

Calculus

$y = x^n$, $y' = nx^{n-1}$,

$y = cf(x)$, $y' = cf'(x)$,

$$If \ y = u \pm v, then \frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$If \ y = uv, then \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$If \ y = \frac{u}{v}, \ then \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int uv\, dx = u \int vdx - \int \left(\frac{du}{dx} \int vdx \right) dx.$$

$$\int_a^b f(x)dx = \lim_{h \to 0} h \left[\sum_{r=0}^{n-1} f(a + rh) \right]$$

$$\frac{dy}{dx} + py = Q, I.F = ye^{\int pdx},$$

$$general\ solution, y.IF = \int (Q.IF)dx + c$$

Data and Probability

$$\overline{X} = \frac{\sum fx}{\sum f} \qquad or \ \overline{X} = \frac{\sum x}{n}$$

$$Median = L + \frac{i}{f}\left(\frac{N}{2} - c \right)$$

$$\overline{X} = \frac{n_1 \overline{x}_1 + n_2 \overline{x}_2}{n_1 + n_2}$$

$$Mean\, Deviation = \frac{\sum |dx_i|}{n}, \quad |dx_i| = |x_i - M|$$

$$\sigma_{12} = \sqrt{\frac{n_1 \sigma_1^{\ 2} + n_2 \sigma_2^{\ 2} + n_1 d_1^{\ 2} + n_2 d_2^{\ 2}}{n_1 + n_2}}$$

$$r = 1 - \frac{6\sum d^2}{n(n^2 - 1)}, \quad Correction\ factor = \frac{1}{12}(m^3 - m)$$

$$r = \pm \sqrt{b_{xy}.b_{yx}}$$

$$b_{xy} \times b_{yx} = r \frac{\sigma_x}{\sigma_y} \times \ r \frac{\sigma_y}{\sigma_x}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A) + P(\overline{A}) = 1 \qquad P\left({}^{B}\!/_{A} \right) = \frac{P(A \cap B)}{P(A)}$$

$$P\left({}^{A}\!/_{B} \right) = \frac{P(A \cap B)}{P(B)}$$

ROUGH WORK

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