SECTION A [30 MARKS]

ANSWER **ALL** QUESTIONS

Question 1 **[30]**

For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than ONE alternative. If there are more than ONE choice circled, NO score will be awarded.

i) The distance between the points $(0,3,4)$ and $(1,-2,5)$ is

A $3\sqrt{3}$ units.

B $\sqrt{83}$ units.

C $\sqrt{3}$ units.

D 9 units.

ii) The sum of $\begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$ and its multiplicative inverse is

A $\begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}$.

B $\begin{bmatrix} 0 & 10 \\ 2 & 0 \end{bmatrix}$.

C $\begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix}$.

D $\begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$.

iii) The amount of annuity immediate of Nu 1500 per annum for 10 years with the interest rate of 5% p.a. is

A Nu 17,867.

B Nu 18,678.

C Nu 18,786.

D Nu 18,867.

iv) If $f(x)=6x^2-4x$ and $f'(a)=10$, then the value of a is

A $\frac{7}{6}$.

B $\frac{6}{7}$.

C 560.

D 600.

v) If $\begin{bmatrix} x^2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} -4x & -4 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ -1 & 5 \end{bmatrix}$, then the value of x that satisfies the matrix equation is

A $-2.$

B $-4.$

C $2.$

D $4.$

vi) Bhutan Agro Industry finds that the total cost of producing and marketing chilli pickles is $C(x) = 100x^2 + 75x - 25000$. Each product is sold at Nu 150. The profit function of producing chilli pickles is

A $P(x) = -100x^2 + 75x - 25000.$

B $P(x) = -100x^2 + 75x + 25000.$

C $P(x) = 100x^2 - 75x - 25000.$

D $P(x) = 100x^2 - 75x + 25000.$

vii) The value of $\int \frac{1}{5-4x} dx$ is

A $\frac{4}{(5-4x)^2} + c.$

B $-4\log(5-4x) + c.$

C $\frac{1}{4}\log(5-4x) + c.$

D $-\frac{1}{4}\log(5-4x) + c.$

viii) If $\begin{vmatrix} x & 0 & x^2-1 \\ 0 & 2 & -1 \\ -1 & 0 & x \end{vmatrix} = 2$, then the value of x is

A $0.$

B $\pm 1.$

C $\pm 2.$

D $\pm 4.$

ix) The total revenue received from the sale of x units of the product is given by $R(x) = 4x^2 - 2x + 6$.

When 10 units are sold, marginal revenue is Nu 78.
When 15 units are sold, marginal revenue is Nu 118.

What will be the marginal revenue if 20 units are sold?

A Nu 78
B Nu 18
C Nu 158
D Nu 160

x) Differentiation of $x^2 + 1$ w.r.t $1 - \frac{2}{x}$ is

A $-x^3$.
B $-\frac{4}{x}$.
C x^3.
D $\frac{4}{x}$.

xi) Given that $P(A) = \frac{2}{5}$, $P(A/B) = \frac{3}{5}$ and $P(A \cap B) = \frac{1}{5}$, then $P(B/A)$ is

A $\frac{1}{5}$.
B $\frac{3}{10}$.
C $\frac{2}{5}$.
D $\frac{1}{2}$.

xii) If the value and annual rent of perpetuity are Nu 120,000 and Nu 10,500 respectively, then the rate of compound interest will be

A 8.75%.
B 9.75%.
C 10.75%.
D 11.43%.

xiii) The profit function of producing x units of pencil is given by $P(x) = 50 + 800x - 2x^2$.

How many pencils should the company sell to make a maximum profit?

A 20
B 200
C 2000
D 80050

xiv) For 4 observations of pairs of variables ***x*** and ***y***, the following results are obtained:

$\sum x = 12, \sum y = 16$, $b_{yx} = \frac{3}{4}$ and $b_{xy} = \frac{2}{3}$. The equation of regression line of y on x is

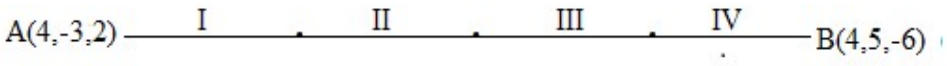
A $3y - 2x - 6 = 0.$

B $3y + 2x + 6 = 0.$

C $4y + 3x + 7 = 0.$

D $4y - 3x - 7 = 0.$

xv) The line segment joining the points $A(4,-3,2)$ and $B(4,5,-6)$ is divided into four equal parts as shown below:



What is the coordinate of a point which is three-fourth of the way from A to B?

A $\left(4, \frac{3}{7}, -\frac{10}{7}\right)$

B $(-4,-9,10)$

C $(4, 3, -4)$

D $(-4,-3, 4)$

SECTION B [70 MARKS]
ATTEMPT **ANY TEN** QUESTIONS

Question 2

a) A man buys a car worth Nu 850,000. He agrees to pay Nu 350,000 immediately and the balance amount in 60 equal monthly installments with 12% p.a. compounded monthly. What is the amount of the monthly installments? **[3]**

b) Determine whether the system of linear equations given below is consistent using matrix method. If consistent, write the solution of the system of equations. **[4]**

$$2x - 3y = -1$$
$$-4x + 6y = 2$$

Question 3

a) If $x = t^2 + 1$ and $y = 2t^3 + t^2$, find $\frac{dy}{dx}$. **[3]**

b) Lines L_1 and L_2 are perpendicular to each other and if L_1 passes through the points $(2, 3, -4)$ and $(-3, 3, x)$ and L_2 passes through $(-1, 4, 6)$ and $(3, 5, 1)$, what is the value of x ? **[4]**

Question 4

a) Due to the pandemic, Ms. Pema started producing face masks and found out that the production cost of each face mask is Nu 30 and the fixed expenditure of production was Nu 25,000. If each face mask is sold for Nu 45, determine **[3]**

i. cost and revenue function.
ii. average cost function.
iii. marginal average cost function.

b) If the two regression lines are $2x + 3y = 6$ and $x + y = 8$, find the correlation coefficient between x and y. Also estimate the value of x when $y = 5$. **[4]**

Question 5

a) Integrate $\int \left(x^2 + x + 1\right)^5 (2x+1)dx.$ **[3]**

b) Given $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 3 & 2 \end{bmatrix}$, is it possible to compute AB? If possible, compute the product AB and find the Minors M_{11} and M_{23} of the matrix AB. **[4]**

Question 6

a) Mr. Tashi takes a loan of Nu 500,000 from T-Bank on the condition to repay it with a compound interest rate of 9% p.a. with a monthly installment of Nu 10,300. In how many installments will the debt be repaid? **[3]**

[4]

b) For the given matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -3 & 4 \\ 0 & 5 & -2 \end{bmatrix}$, verify that $A(AdjA) = |A| I_3$.

Question 7

a) Calculate the rank coefficient of correlation for the given data. **[3]**

x	15	20	12	40	40	35
y	40	30	30	20	70	50

b) The pricing policy of Dungsam Cement Corporation Limited follows the demand function $p = D(x)$, $D(x)$ being the price per bag of cement when x units are demanded. After studying the market trends, the company determines the price function as $D(x) = 4750 - 20x$. For the production, the fixed cost is Nu 250,000 and cost of producing a bag of cement is Nu 250. How many bags of cement should the company sell to recover its total cost? **[4]**

Question 8

a) Given homogenous system of equations is consistent. Use the condition for consistency of homogenous equation to determine the value of k. **[3]**

$$2x + 3y - 17 = 0$$
$$x - 2y + 16 = 0$$
$$kx + y - 1 = 0$$

b) Evaluate $\int \frac{2x^3 - 4x^2 + 3x + 1}{x + 1} dx$. **[4]**

Question 9

a) If $y = \left(2x^3 + 1\right)\left(4x^2 - 3\right)$, prove that $\frac{d^2y}{dx^2} = 160x^3 - 36x + 8$. **[3]**

b) Mr. Jigme buys 5 acres of land paying Nu 3,000,000 and promises to pay Nu 50,000 twice a year for the next 5 years. The interest is 10% p.a. compounded semi-annually. If Mr. Jigme misses the first 4 installments, what will be the total payment at the time of the 5^{th} installment? **[4]**

Question 10

a) Prove that $\begin{vmatrix} 1 & a & x \\ 1 & x & a \\ 1 & x & x \end{vmatrix} = (x-a)^2$. **[3]**

b) Find the Karl Pearson's correlation coefficient between the marks awarded by 5 judges to two singers A and B during the final round of Voice of Bhutan. Also interpret the result. The marks awarded by the five judges are as follows: **[4]**

Singer A	10	25	13	15	22
Singer B	12	22	16	9	11

Question 11

a) Find the angle between the lines whose direction ratios are $(2,1,3)$ and $(-1,5,3)$. **[3]**

b) If $3x^2 + 2y - 2xy = 4$, find $\frac{dy}{dx}$ at $(-1, 2)$. **[4]**

Question 12

a) At the beginning of each quarter, Nu 15,000 is deposited in the savings account that pays an interest at the rate of 12% p.a. compounded quarterly. Find the balance in the account at the end of 3 years. [Given $(1.03)^{12} = 1.426$] **[3]**

b) Resolve $\frac{4x+1}{(x+1)^2(x+2)}$ into partial fraction and integrate the given function. **[4]**

Question 13

a) If the distance between the points $(x,-1,3)$ and $(1,-3,1)$ is 3 units, prove that abscissa is 2 or 0. **[3]**

b) The average cost function for a commodity is given by $AC = x + 4 + \frac{16}{x}$ in terms of output x. Find the total cost and marginal cost as function of x. Also find the output for which AC decreases. **[4]**

Question 14

a) A box contains 6 blue balls, 5 red balls and 4 green balls. Two balls are drawn randomly from a box. What is the probability that **[3]**

i. both are blue?

ii. both are red?

iii. none of them are green?

b) A cable service provider in a town has 400 subscribers and collects fixed charges of Nu 300 per subscriber per year. The cable service provider proposed to increase the annual subscription fee and it is believed that for every increase of Nu 1, one subscriber will discontinue the service. What increase in subscription fee will bring maximum income to the cable service provider? Also find the maximum income of the cable service provider. **[4]**

MATHEMATICS FORMULAE

Co-ordinate Geometry

$$D=\sqrt{(x_2-x_2)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$(x,y,z)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2},\frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$$

$$a_1x+b_1y+c_1z=0 \text{ and } a_2x+b_2y+c_2z=0$$

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{z}{a_1b_2-a_2b_1}$$

$$\cos\theta=\pm\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Algebra

$$a^2-b^2=(a+b)(a-b),\quad (a\pm b)^2=a^2\pm 2ab+b^2$$

$$\text{In the Q.E } ax^2+bx+c=0,\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

,

$$C_{ij}=(-1)^{i+j}M_{ij},$$

$$AA^{-1}=A^{-1}A=I$$

$$A^{-1}=\frac{1}{\det A}\cdot adjA$$

$$x=\frac{D_x}{D},y=\frac{D_y}{D},z=\frac{Dz}{D}$$

Commercial Mathematics

$$A=\frac{a}{i}(1+i)\left[(1+i)^n-1\right],$$

$$P=\frac{a}{i}\left[1-(1+i)^{-n}\right]$$

$$AC=\frac{C(x)}{x},\ MC=\frac{d}{dx}(C)$$

$$C(x)=F+V(x),\qquad R(x)=xP(x)$$

$$P(x)=R(x)-C(x)$$

Calculus

$$y=x^n,\ \ y'=nx^{n-1}$$

$$y=cf(x),\ y'=cf'(x),$$

$$If\ y=u\pm v, then\ \frac{dy}{dx}=\frac{du}{dx}\pm\frac{dv}{dx}$$

$$If\ y=uv, then\ \frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$$

$$If\ y=\frac{u}{v},\ then\ \frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$$

Data & Probability

$$r=\frac{n\sum xy-\sum x\sum y}{\sqrt{n\sum x^2-(\sum x)^2}\sqrt{n\sum y^2-(\sum y)^2}},$$

$$r=1-\frac{6\sum d^2}{n(n^2-1)},\ r=\frac{\sum d_xd_y}{n\sigma_x\sigma_y},\ r=\sqrt{b_{yx}b_{xy}}$$

$$b_{xy}=r\frac{\sigma_x}{\sigma_y}=\frac{n\sum xy-\sum x\sum y}{n\sum y^2-(\sum y)^2}$$

Rough Work

Rough Work

Rough Work