SECTION A [30 MARKS]

ANSWER **ALL** QUESTIONS

Question 1 **[30]**

Direction: For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than ONE alternative. If there are more than ONE choice circled, NO score will be awarded.

i) What is the adjoint of the matrix $\begin{bmatrix} 2 & -3 \\ 0 & -5 \end{bmatrix}$?

A $\begin{bmatrix} 2 & 0 \\ -3 & -5 \end{bmatrix}$

B $\begin{bmatrix} -5 & 3 \\ 0 & 2 \end{bmatrix}$

C $\begin{bmatrix} -5 & 0 \\ -3 & 2 \end{bmatrix}$

D $\begin{bmatrix} -2 & -3 \\ 0 & 5 \end{bmatrix}$

ii) Find the principle value of $\sin\left(\tan^{-1}1\right)+\cos\left(\sec^{-1}2\right)$.

A $\frac{1}{2}$

B $\frac{2}{\sqrt{2}+1}$

C 1

D $\frac{\sqrt{2}+1}{2}$

iii) Find the minimum value of $\frac{1}{x}+x$.

A 2

B 1

C -1

D -2

iv) What are the direction cosines of a line from $\mathrm{A}(4,-4,-2)$ to the origin?

A $\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}$

B $-2, 2, 1$

C $\frac{2}{3}, \frac{-2}{3}, \frac{-1}{3}$

D $2, -2, -1$

v) The regression coefficient of x on y is 3.2. Which of the following value is the possible regression coefficient of y on x?

A 1

B 0.8

C 0.25

D -1

vi) The amplitude of $-\sqrt{3}+i$ is

A $\frac{7\pi}{6}$.

B $\frac{5\pi}{6}$.

C $\frac{\pi}{6}$.

D $\frac{-5\pi}{6}$.

vii) Find the value of x if $\begin{vmatrix} x & 0 & 0 \\ 3 & 1 & 2 \\ 4 & 1 & 0 \end{vmatrix} = 4$.

A 4

B 2

C -2

D -4

viii) Find the length of the latus rectum of the equation $x^2 = -10y$.

A 10

B $\frac{5}{2}$

C $\frac{-5}{2}$

D -10

ix) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ is

A $2e^{\sqrt{x}} + C$.

B $\frac{e^{\sqrt{x}}}{\sqrt{x}} + C$.

C $\frac{e^{\sqrt{x}}}{2\sqrt{x}} + C$.

D $\frac{1}{2\sqrt{x}} + C$.

x) Match list-I (Equality/Inequality) with list-II (Inference) and select the correct answer.

List-I (Equality/Inequality)	List-II (Inference)
I. $P(E_1)+P(E_2)=1$	1. E_1, E_2 are mutually exclusive events
II. $P(E_1)+P(E_2)=0$	2. E_1, E_2 are mutually exhaustive events
III. $P(E_1)+P(E_2)\le 1$	3. E_1, E_2 are not sure events
IV. $P(E_1).P(E_2)=1$	4. E_1, E_2 are impossible events
	5. E_1, E_2 are not equally likely events

	I	II	III	IV
A	2	1	4	3
B	3	1	5	2
C	2	4	1	3
D	1	3	2	5

xi) Find the derivative of $\cos^2\left(x^2\right)$.

A $2\sin 2x^2$

B $x\sin 2x^2$

C $2x\sin 2x^2$

D $-2x\sin 2x^2$

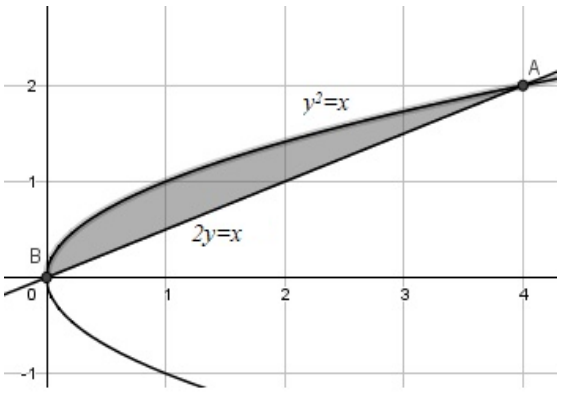
xii) The area of the shaded region of the figure is

A $\frac{1}{3}$ sq. unit.

B $\frac{1}{2}$ sq. unit.

C $\frac{2}{3}$ sq. unit.

D $\frac{4}{3}$ sq. unit.



xiii) The equation of the plane parallel to $y-$axis such that the $x-$intercept is equal to -3 and the $z-$intercept is equal to 4 is

A $-3x+4z+12=0.$

B $4x-3z+12=0.$

C $3x-4z=0.$

D $y=0.$

xiv) Find $\frac{dy}{dx}$ if $x=a\cos t$ and $y=a\sin t.$

A $\tan t$

B $\cot t$

C $-\tan t$

D $-\cot t$

xv) Evaluate the value of λ so that $x^2+4xy+4y^2+\lambda x+10y+4=0$ represents a pair of straight lines.

A 1

B 2

C 4

D 5

SECTION B [70 MARKS]

ATTEMPT **ANY TEN** QUESTIONS

Question 2

a) If $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + ...\infty}}}$, prove that $(2y-1)\frac{dy}{dx} - \sec^2 x = 0.$ **[3]**

b) Using the properties, prove that $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$. **[4]**

Question 3

a) Find the equation of the ellipse whose foci $(\pm 5,\ 0)$ and the length of the semi-minor axis is 12 units. **[3]**

b) The following marks were obtained by 7 students in Mathematics and Physics. Find the regression equation of y on x. **[4]**

Mathematics	5	7	8	4	6	5	7
Physics	2	4	3	2	4	4	2

Question 4

a) Solve the following system of equations using matrix method. **[4]**

$$x - 2y + z = 0$$
$$y - z = 2$$
$$2z - 3z = 10$$

b) Find the real numbers x and y, if $(x - yi)(2 + 3i)$ is a conjugate of $-4 - 20i$. **[3]**

Question 5

a) If the region bounded by the curve $y = x^2$ and the lines $x = 0,\ x = 2$ is rotated through four right angles about x-axis, calculate the volume of the solid so formed. **[3]**

b) Solve the following equations:

i) $\tan\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$ **[2]**

ii) $\cos\left(\sin^{-1} x\right) = \frac{4}{5}$ **[2]**

Question 6

a) Find 2×2 matrix such that **[3]**

$$X\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}.$$

b) Show that the four points $(0,\ 4,\ 3), (-1,\ -5,\ -3), (-2,\ -2,\ 1)$ and $(1,\ 1,\ -1)$ are coplanar and find the equation of the common plane. **[4]**

Question 7

a) Prove that $\tan\left[\cos^{-1}\left\{\cot\left(\sin^{-1}x\right)\right\}\right]=\frac{1}{\sqrt{1-x^2}}$. **[3]**

b) A bowl contains 4 red, 6 white and 10 blue paper strips. If 4 paper strips are drawn one by one without replacement, find the probability of getting all four strips of the same colour. **[4]**

Question 8

a) Find the value of k, if the following equations are consistent. **[2]**

$2x + 3y - 17 = 0, \qquad x - 2y + k = 0, \qquad 3x + y - 5 = 0$

b) Differentiate the following functions:

i) $y = \text{cosec}^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \cos^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$ **[2.5]**

ii) $y = \log_4 \sin x$ **[2.5]**

Question 9

a) Find the equation of the bisector of the angles between the pair of straight lines represented by the equation $4x^2 + 6xy - 2y^2 = 0$. **[3]**

b) Find the locus of a complex number $z = x + yi$ satisfying the relation $\left|\frac{z-3i}{z+3i}\right| = \sqrt{3}$. **[4]**

Illustrate the locus of z in the Argand diagram.

Question 10

a) The fuel price at Bumthang is Nu 60. Calculate the fuel price at Gelephu, if their correlation coefficient is 0.6 with the help of the following table: **[3]**

Places	**Gelephu**	**Bumthang**
Average price	42	55
Standard deviation	1.5	2.5

b) Evaluate: $\int \frac{2x-3}{x^2-3x-4}\,dx$ **[4]**

Question 11

a) Following are the ranks awarded by an Agriculture Extension Officer to 6 farmers after assessing their dairy and poultry farms. To what extent are the quality of their farms related to each other? **[3]**

Dairy	1	2	3	4	5	6
Poultry	2	3	1	4	6	5

b) Find the equation of a plane which passes through $(-1,\ -1,\ 2)$ and is perpendicular to the planes $3x + 2y - 3z = 1$ and $5x - 4y + z = 5$. **[4]**

Question 12

a) Evaluate: $\int_0^2 \frac{2}{3x^2+4}\,dx$ **[4]**

b) Mr. Thinley designs an arc bridge with computer simulations represented by $7x^2 + 3y^2 - 28x + 24y + 55 = 0$. Which conic does the arc represent? **[3]**

Question 13

a) Evaluate: $\int \frac{x+\frac{1}{\cos ecx}}{1+\frac{1}{\sec x}}dx$ **[4]**

b) If α, β, γ are the angles that a line makes with the axes, find $\cos\alpha$, if **[3]**

$\cos\beta = \frac{1}{2}$ and $\cos\gamma = \frac{-1}{\sqrt{2}}$.

Question 14

a) Differentiate $\frac{\sin^2 x(x^2 - 2x + 1)}{e^{x+2}}$. **[4]**

b) If the coordinates P and Q be $(1,2,3)$ and $(2,-7,3)$ respectively. Prove that OA is perpendicular to OB, where O is the origin. **[3]**

MATHEMATICS FORMULAE

Trigonometry

$$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

$$\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right)$$

$$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left(\frac{x \pm y}{1 \mp xy}\right), xy < 1$$

$$2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2} = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}$$

$$\cos ec^{-1} x = \sin^{-1}\frac{1}{x}$$

$$\sec^{-1} x = \cos^{-1}\frac{1}{x}$$

$$\cot^{-1} x = \tan^{-1}\frac{1}{x}$$

Complex Numbers

$$r = \sqrt{a^2 + b^2}$$

$$\tan\theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\left|\frac{b}{a}\right|$$

$$z = r(\cos\theta + i\sin\theta)$$

Co-ordinate Geometry

$a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

Angle between two lines

$$\cos\theta = \pm\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

equation of bisector: $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

points of intersection: $\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$

Algebra

$$a^2 - b^2 = (a+b)(a-b)$$

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

In the Q E : $ax^2 + bx + c = 0,\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$A\,A^{-1} = A^{-1}A = I$$

$$A^{-1} = \frac{1}{\det A} \cdot adjA$$

$$x = \frac{D_x}{D}, y = \frac{D_y}{D}, z = \frac{Dz}{D}$$

$$C_{ij} = (-1)^{i+j} M_{ij}$$

Calculus

$y = x^n,\ y' = nx^{n-1},$

$y = cf(x),\ y' = cf'(x),$

$$1 + 2 + 3 + ... + (n-1) = \frac{1}{2}n(n-1)$$

$$1^2 + 2^2 + 3^2 + ... + (n-1)^2 = \frac{1}{6}n(n-1)(2n-1)$$

$$1^3 + 2^3 + 3^3 + ... + (n-1)^3 = \left\{\frac{n(n-1)}{2}\right\}^2$$

$$If\ y = u \pm v, then\ \frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$If\ y = uv, then\ \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$If\ y = \frac{u}{v},\ then\ \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int uv\,dx = u\int v dx - \int\left(\frac{du}{dx}\int v dx\right)dx.$$

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[\sum_{r=0}^{n-1} f(a + rh)\right]$$

$$V = \pi\int_a^b y^2 dx \quad A = \int_a^b y dx$$

Data and Probabilty

$$r = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

$$r = \frac{\sum(x - \bar{x}) - \sum(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2 \sum(y - \bar{y})^2}}$$

$$r = 1 - \frac{6\sum D^2}{n(n^2 - 1)} \quad r = \pm\sqrt{b_{yx} \times b_{xy}}$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - (\sum x)^2}$$

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A) + P(\bar{A}) = 1$$

$$P\left(B/A\right) = \frac{P(A \cap B)}{P(A)}$$

Rough Work

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