SECTION A [30 MARKS]
ANSWER **ALL** QUESTIONS

Question 1 **[30]**

Direction: For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than ONE alternative. If there are more than ONE choice circled, NO score will be awarded.

i) The following system of equations has

$$x + 3y = 0$$
$$2x - y = 0$$

A no solution.
B trivial solution.
C unique solution.
D infinitely many solutions.

ii) The given equation represents

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

A horizontal hyperbola.
B conjugate hyperbola.
C horizontal ellipse.
D vertical ellipse.

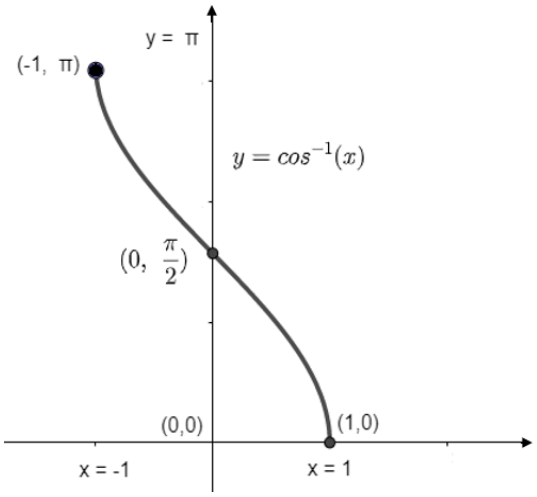
iii) The velocity of a car moving in a straight line at time t is described by $v = 12t(1+t)$ m/s. The acceleration at time $t = 2$ seconds is

A -60 m/s^2.
B -30 m/s^2.
C 30 m/s^2.
D 60 m/s^2.

iv) The order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \sqrt[3]{\frac{dy}{dx}}$ is

A Order: 1, Degree: 2.
B Order: 1, Degree: 3.
C Order: 2, Degree: 2.
D Order: 2, Degree: 3.

v) Identify domain and range from the given figure.



A $\text{Domain}: 0 \le x \le \pi,\ \text{Range}: -1 \le y \le 1$

B $\text{Domain}: 0 \le y \le \pi,\ \text{Range}: -1 \le x \le 1$

C $\text{Domain}: -1 \le x \le 1,\ \text{Range}: 0 \le y \le \pi$

D $\text{Domain}: -1 \le y \le 1,\ \text{Range}: 0 \le x \le \pi$

vi) If omega (ω) is a cube root of unity, find the value of $\left(1+\omega-\omega^2\right)^3 \omega^{21}$.

A -2

B -8

C $-2\omega^2$

D $2\omega^2$

vii) The current license plate for private vehicles in Bhutan is composed of letters and digits in **BP-R-ANNNN** format, where R can be digits from 1 to 5, A is an alphabet and N is a digit. How many number plates can be formed?

A 117000

B 260000

C 655000

D 1300000

viii) What is the value of $\sin^{-1}\frac{1}{2}+2\cot^{-1}\sqrt{3}$?

A $\frac{\pi}{6}$

B $\frac{\pi}{3}$

C $\frac{\pi}{2}$

D $\frac{2\pi}{3}$

ix) The value of $\int \frac{2x-3}{\sqrt{x^2-3x+5}}dx$ is

A $2(x^2-3x+5)^{1/2}+c.$

B $2(x^2-3x+5)+c.$

C $2(x^2-3x-5)^{1/2}+c.$

D $2(x^2-3x-5)+c.$

x) Match column I with column II and select the correct answer.

Colum I	Column II
1. $y=f(t)$ and $x=f(t)$	A. $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{ds}\times\frac{ds}{dx}$
2. $y=f(t),\ t=f(s),\ s=f(x)$	B. $\frac{dp}{dq}=\frac{dp/dx}{dq/dx}$
3. $p=f(x)$ and $q=f(x)$	C. $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$

A 1 – C, 2 – B, 3 – A

B 1– C, 2 – A, 3 – B

C 1 – B, 2 – C, 3 – A

D 1 – A, 2 – B, 3 – C

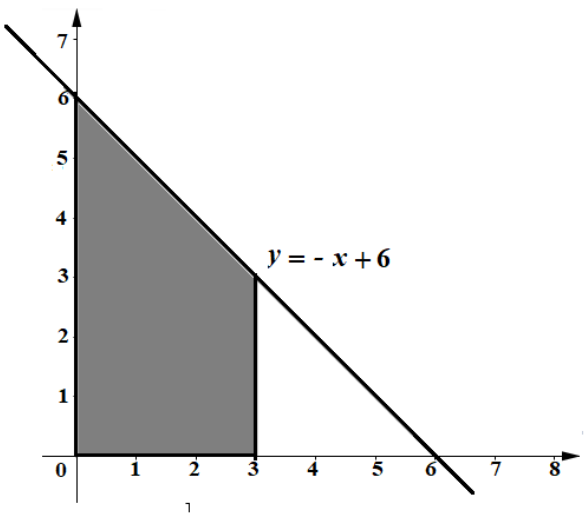
xi) The area of the shaded region in the given figure is

A $\frac{3}{2}$ sq. units.

B $\frac{9}{2}$ sq. units.

C $\frac{27}{2}$ sq. units.

D $\frac{45}{2}$ sq. units.



xii) The angle between two lines whose direction cosines are $\frac{1}{2},\frac{1}{2},-1$ and 1, 2, 1 is

A 60^0.

B 90^0.

C 120^0.

D 180^0.

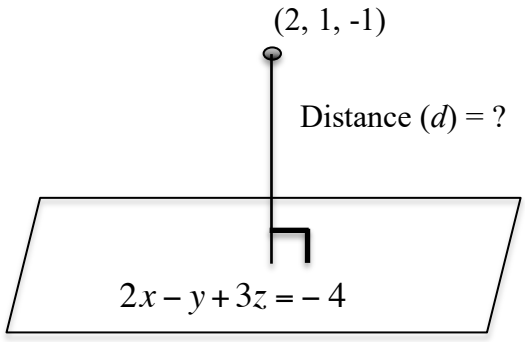
xiii) Find the distance (d) of the point from the plane.

A $\frac{2\sqrt{14}}{7}$ units

B $\frac{\sqrt{14}}{7}$ units

C $\frac{2}{\sqrt{7}}$ units

D $\frac{\sqrt{14}}{2}$ units



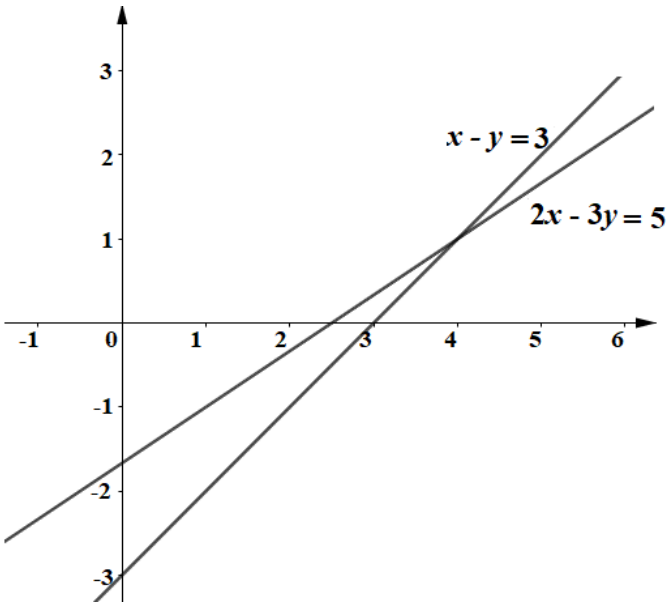
xiv) Find the correlation coefficient between x and y.

A $\frac{\sqrt{2}}{3}$

B $\frac{2}{\sqrt{3}}$

C $\sqrt{\frac{2}{3}}$

D $\sqrt{\frac{3}{2}}$



xv) An anti-aircraft gun can take a maximum of two shots at an enemy plane moving away from it. The probability of hitting the plane at the first and second shots is 0.3 and 0.2 respectively. What is the probability that the shot will hit the plane?

A 0.80
B 0.70
C 0.56
D 0.44

SECTION B [70 MARKS]
ATTEMPT **ANY TEN** QUESTIONS

Question 2

a) Evaluate: $\int_1^4 |x-2|\,dx$ **[3]**

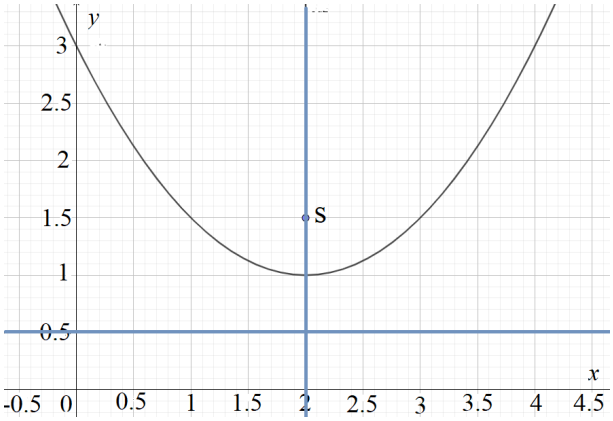
b)

i) State the type of permutation for each given statement. **[2]**
A) Arrangement of different flowers to form a garland.
B) Arrangement of letters of the word **PELELA**.

ii) There are 12 house captains competing for the post of school captains. In how many ways can 3 captains be selected such that one particular captain is always selected? **[2]**

Question 3

a) Using the information from the given figure, find the equation of the curve. **[3]**



b) Prove that $\tan\left(\sin^{-1}\frac{4}{5}-\cos^{-1}\frac{1}{\sqrt{5}}\right)=-\frac{2}{11}$. **[4]**

Question 4

a) Find $\frac{dy}{dx}$, if $e^{\log \sin x} = xy$. **[3]**

b) Following are the marks scored by 8 students in their first unit test in Mathematics and Physics. Calculate the correlation coefficient between these subjects and interpret the result.

Mathematics	75	29	16	27	71	51	58	86
Physics	36	31	32	15	42	39	48	50

[4]

Question 5

a) Determine the value of β if $6x^2 + xy + \beta y^2 - 11x + 43y - 35 = 0$. **[3]**

b) In a school library there is a question bank for Science which contains 60 easy multiple choice questions (MCQ) and 40 difficult MCQ, 120 easy True/False questions and 80 difficult True/False questions. If a question is selected at random from the question bank, what is the probability that it will be a difficult MCQ? **[4]**

Question 6

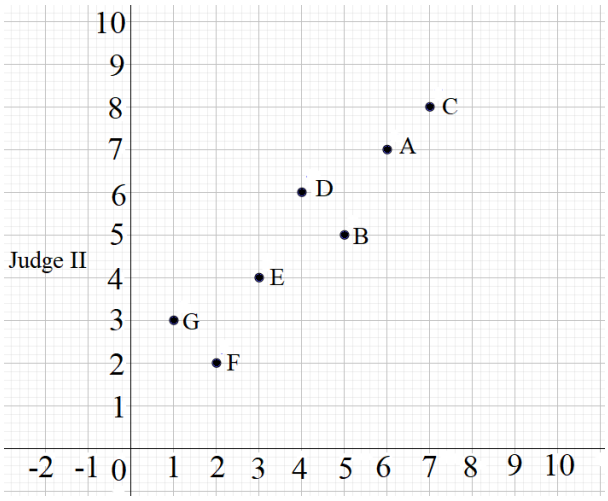
a) Sonam invests Nu 100,000 in two types of bonds. The first bond pays 6% interest p.a. and the second pays 6.5% interest p.a. Using matrix multiplication, determine the amount he needs to invest in those two bonds in order to earn an interest amount of Nu 6350. **[3]**

b) Find the equation of the plane passing through points (4, -2, 3) and (0, 2, -3), if the sum of their intercepts on the three axes is zero. **[4]**

Question 7

a) Find $\frac{d^2y}{dx^2}$ if $y = \tan^{-1} t$ and $x = e^{\tan^{-1} t}$. **[3]**

b) Gelephu Thromde has conducted a cultural dance competition amongst seven schools (A, B, C, D, E, F, G). The scatter plot given below shows the scores awarded by two judges.



i) Compute the rank correlation coefficient between the two judges. **[1.5]**

ii) If the score of judge II for school B is changed to 1, how will the rank correlation coefficient between the two judges change? **[2.5]**

Question 8

a) Find the integral of $\frac{x}{\sqrt{x+2}}$. **[2]**

b) Solve $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1.$ **[5]**

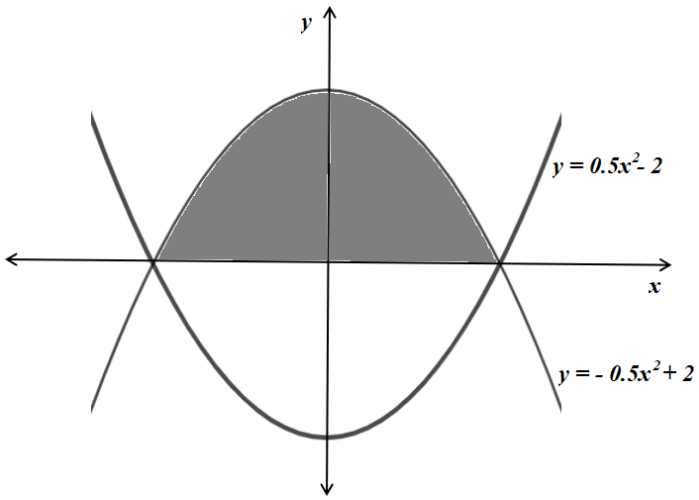
Question 9

a) Find the eccentricity, length of latus rectum and coordinates of foci of the conic section $3x^2 - 2y^2 = 24$. **[3]**

b) Solve the differential equation $\frac{d^2y}{dx^2} = \frac{1}{x}$, given $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 1$. **[4]**

Question 10

a) Find the volume of solid of revolution formed when the shaded portion in the given figure is rotated about the x-axis. **[3]**

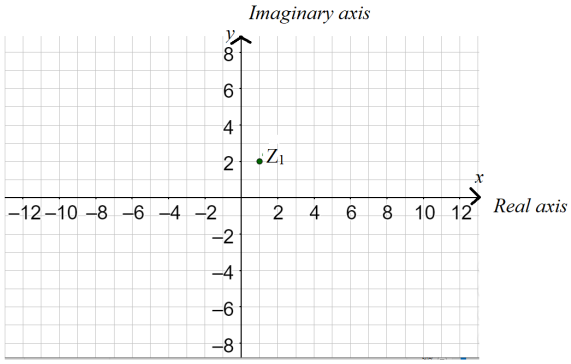


b) Show that $3x^2 - 3y^2 + 8xy + 5x + 5y + 2 = 0$ represents pair of perpendicular straight lines. **[4]**

Question 11

a) From the equation of the ellipse $9x^2 + 4y^2 + 6x - 24y + 36 = 0,$ find the centre and semi axes. **[3]**

b) Illustrate in the complex plane the set of points represented by $|z+z_1-2|\leq 3$. Explain your answer. **[4]**



Question 12

a) Twenty metres of wire is available to fence a flower bed in the form of a sector. What would be the value of radius, if the area of the flower bed is maximum? **[3]**

b) Sonam purchased 2 pencils and 3 erasers from a shop and paid a total of Nu 23. From the same shop, Penjor purchased 3 pencils and 4 erasers and paid Nu 35. Translate the problem into system of equations and solve using Martin's rule. What is the cost of one pencil and one eraser? **[4]**

Question 13

a) If $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -2 \\ 1 & 2 & -4 \end{bmatrix}$, find $\left(A'\right)^{-1}$. **[3]**

b) Evaluate: $\int \frac{\cos\theta}{\sin^2\theta - 5\sin\theta + 6} d\theta$ **[4]**

Formulae

Strand A : Numbers and Operations

$a^2 - b^2 = (a+b)(a-b)$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

In QE $ax^2 + bx + c = 0, x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

${}^nP_r = \dfrac{n!}{(n-r)!}$

${}^nC_r = \dfrac{n!}{r!(n-r)!}$

$C_{ij} = (-1)^{i+j} M_{ij}$

$A^{-1} = \dfrac{1}{|A|} adjA$

$AA^{-1} = A^{-1}A = I$

$r = \sqrt{a^2 + b^2}$

$\tan\theta = \dfrac{b}{a}, \Rightarrow \theta = \tan^{-1}\left|\dfrac{b}{a}\right|$

$z = r(\cos\theta + i\sin\theta)$

Strand B : Patterns and Algebra

If $y = x^n$ then $y' = nx^{n-1}$

If $y = cf(x)$ then $y' = cf'(x)$

If $y = uv$ then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

If $y = \dfrac{u}{v}$ then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$\int x^n dx = \dfrac{x^{n+1}}{n+1} + c$

$\int (ax+b)^n dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + c$

$\int uv\,dx = u\int v dx - \int\left(\dfrac{du}{dx}\int v dx\right)dx$

If $\dfrac{dy}{dx} + py = Q$ then $I.F = e^{\int p dx}$

General solution:

$y(I.F) = \int Q(I.F.)\,dx + c$

$1 + 2 + 3 + \ldots + (n-1) = \dfrac{1}{2}n(n-1)$

$1^2 + 2^2 + 3^2 + \ldots + (n-1)^2 = \dfrac{1}{6}n(n-1)(2n-1)$

$1^3 + 2^3 + 3^3 + \ldots + (n-1)^3 = \left\{\dfrac{n(n-1)}{2}\right\}^2$

$\int_a^b f(x)dx = \lim_{h\to 0} h\left[\sum_{r=0}^{n-1} f(a+rh)\right]$

$A = \int_a^b y\,dx,\ V = \pi\int_a^b y^2\,dx$

Volume of Cone $= \dfrac{1}{3}\pi r^2 h$

Volume of Sphere $= \dfrac{4}{3}\pi r^3$

Volume of Cylinder $= \pi r^2 h$

S.Area of Cone $= \pi rl + \pi r^2$

S.Area of Sphere $= 4\pi r^2$

S.Area of Cylinder $= 2\pi rh + 2\pi r^2$

Area of sector $= \dfrac{1}{2}r^2\theta$

Strand C : Measurement

$\sin^{-1}x \pm \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right)$,

if $x, y \geq 0$ and $x^2 + y^2 \leq 1$

$\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right)$,

if $x, y > 0$ and $x^2 + y^2 \leq 1$

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$, if $xy < 1$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right)$, if $xy > -1$

$2\tan^{-1}x = \tan^{-1}\dfrac{2x}{1-x^2}$

$= \sin^{-1}\dfrac{2x}{1+x^2} = \cos^{-1}\dfrac{1-x^2}{1+x^2}$

$\text{cosec}^{-1}x = \sin^{-1}\dfrac{1}{x}$

$\sec^{-1}x = \cos^{-1}\dfrac{1}{x}$

$\cot^{-1}x = \tan^{-1}\dfrac{1}{}$

Strand D : Geometry

Angle between two lines

$$\cos\theta = \pm\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

If $a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

$$l = \pm\frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \pm\frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \pm\frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$$|\mathrm{SP}| = e|\mathrm{PM}|$$

$$\Rightarrow \sqrt{(x - x_1)^2 + (y - y_1)^2} = \left|\frac{ax + by + c}{\sqrt{a^2 + b^2}}\right|$$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

Equation to the bisectors of angles:

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

The point of intersection:

$$\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$$

Strand E: Data Management and Probability

$$r = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2\sum(y - \bar{y})^2}}$$

$$r = 1 - \frac{6\sum D^2}{n(n^2 - 1)}$$

$$\text{Correction factor} = \frac{1}{12}\sum(m^3 - m)$$

$$r = \pm\sqrt{b_{yx} \times b_{xy}}$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - (\sum x)^2}$$

$$b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x\sum y}{n\sum y^2 - (\sum y)^2}$$

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A) + P(\bar{A}) = 1$$

$$P(B / A) = \frac{P(A \cap B)}{P(A)}, P(A) \neq 0$$

$$P(A / B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Rough Work

Rough Work