SECTION A [30 MARKS]

ANSWER **ALL** QUESTIONS

Question 1 **[30]**

Direction: For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than ONE alternative. If there is more than ONE choice circled, NO score will be awarded.

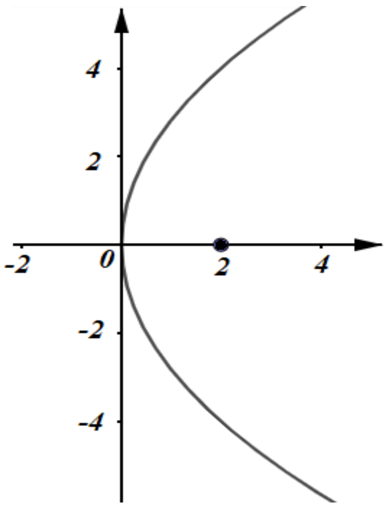
i) If $\tan\left(\tan^{-1} x\right) = x,$ then the values of x are

A $(-1,\ 1)$.
B $(-\infty,\ -1)$.
C $(1,\ \infty)$.
D $(-\infty,\ \infty)$.

ii) X is a $r \times 2$ matrix and Y is a $q \times 3$ matrix. If X and Y are conformable and XY is a 4×3 matrix, what is the value of r and q?

A $r = 2,\ q = 3$
B $r = 2,\ q = 4$
C $r = 3,\ q = 2$
D $r = 4,\ q = 2$

iii) Find the length of latus rectum for the given parabola.



A 16 units
B 8 units
C 4 units
D 2 units

iv) When Dhendup computed the correlation coefficient between the height and weight of 20 students, he found r < 0.5. Interpret the correlation.

A Perfect positive correlation
B High degree positive correlation
C Low degree positive correlation
D Moderate degree positive correlation

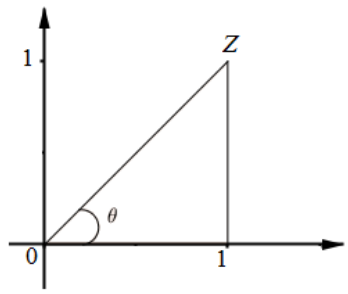
v) Convert the given complex number (z) into polar form.

A $\sqrt{2}\ \text{cis}\frac{\pi}{6}$

B $\sqrt{2}\ \text{cis}\frac{\pi}{4}$

C $\sqrt{2}\ \text{cis}\frac{\pi}{3}$

D $\sqrt{2}\ \text{cis}\frac{\pi}{2}$



vi) If $3x^2 + y^{\frac{3}{2}} = 2$, find $\frac{dy}{dx}$ at $(1,\ 4)$.

A –4
B –3
C –2
D –1

vii) The shortest distance between the planes $2x - y + 3z - 4 = 0$ and $6x - 3y + 9z + 13 = 0$ is

A $\frac{25}{3\sqrt{14}}$ *units*.

B $\frac{1}{3\sqrt{14}}$ *units*.

C $\frac{25}{3\sqrt{2}}$ *units*.

D $\frac{1}{3\sqrt{2}}$ *units*.

viii) In a school soccer team, there are 12 players of whom 2 are teachers. In how many ways can a team of 11 players be selected, so as to include at least one teacher?

A 2
B 6
C 10
D 12

ix) What is the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin 3x dx$?

A 0

B $\frac{1}{3}$

C 3

D ∞

x) The derivative of $\left(2x^3+4x\right)$ w. r. t. $-\frac{2}{x}$ is

A $x^2\left(3x^2+2\right)$.

B $x^2\left(3x^2-2\right)$.

C $x^2\left(3x^2+2x\right)$.

D $x^2\left(3x^2-2x\right)$.

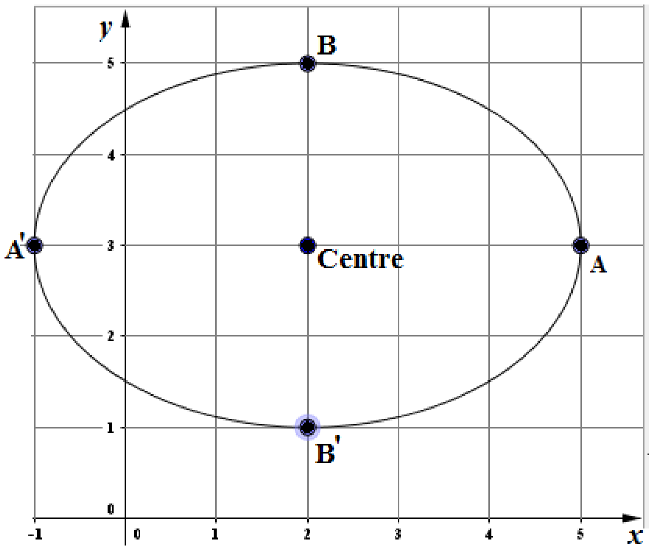
xi) Find the equation of the given ellipse.

A $\frac{(x+2)^2}{4}+\frac{(y+3)^2}{9}=1$

B $\frac{(x+2)^2}{9}+\frac{(y+3)^2}{4}=1$

C $\frac{(x-2)^2}{4}+\frac{(y-3)^2}{9}=1$

D $\frac{(x-2)^2}{9}+\frac{(y-3)^2}{4}=1$



xii) The differential equation $\cos^2 x\, dy = \operatorname{cosec} y\, dx$ has a solution of the form

A $\tan x - \cos y = c$.

B $\tan x + \cos y = c$.

C $\tan y - \cos x = c$.

D $\tan y + \cos x = c$.

xiii) Find the angle between the pair of straight lines represented by $x^2 + 4xy + y^2 = 0$.

A 30°
B 45°
C 60°
D 90°

xiv) A Chemistry problem is given to two students A and B. Their chances of solving the problem are $\frac{3}{4}$ and $\frac{2}{5}$ respectively. What is the probability that one of the students solves the problem?

A $\frac{17}{20}$
B $\frac{11}{20}$
C $\frac{7}{20}$
D $\frac{3}{20}$

xv) If $f\,'(x) = 6x^5 - \frac{3}{x^4}$ and $f(1) = 4$, then $f(x)$ is

A $x^6 + \frac{1}{x^3} - 2.$
B $x^6 - \frac{1}{x^3} + 2.$
C $x^6 + \frac{1}{x^3} + 2.$
D $x^6 - \frac{1}{x^3} - 2.$

SECTION B [70 MARKS]

ATTEMPT **ANY TEN** QUESTIONS

Question 2

a) Write down the value of *r* for each figure and specify the degree of correlation between two regression lines. **[3]**

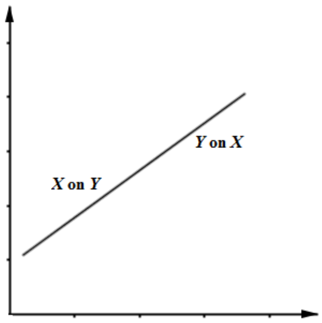


Fig. 1

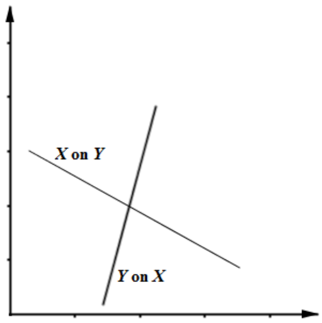


Fig. 2

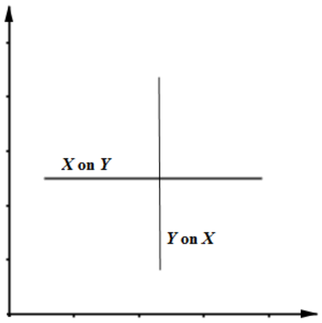
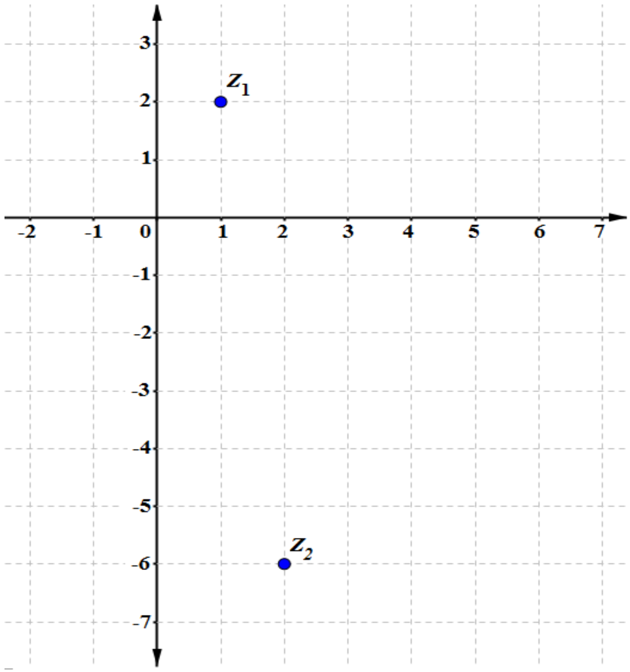


Fig. 3

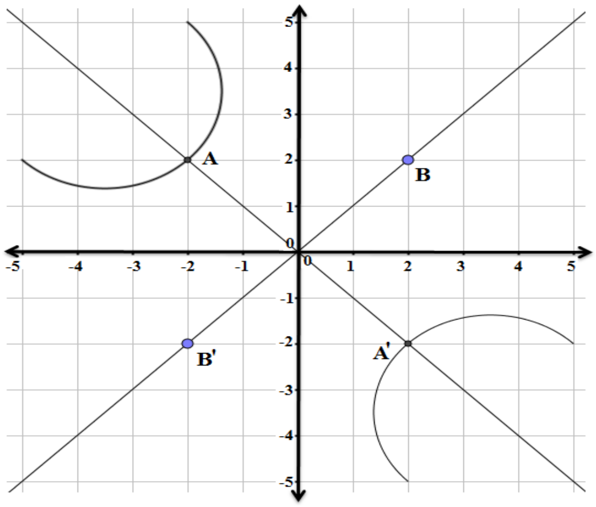
b) From the complex plane, find the square root of $z_1 + z_2$. **[4]**



Question 3

a) Evaluate: $\int \frac{1}{\sqrt{3x+4}-\sqrt{3x+1}}\,dx$ **[4]**

b) Find the length of major axis, length of minor axis and the value of eccentricity for the conic section. **[3]**



Question 4

a) In an annual cluster sports meet, 6 students competed in both high jump (x) and long jump (y) and the following results were obtained.
$\sum x = 14,\ \sum y = 23,\ \sum x^2 = 36.5,\ \sum y^2 = 99,\ \sum xy = 57.$
Find the regression equation y on x and estimate the long jump distance of a student whose high jump distance is 7 m. **[4]**

b) If $B = \begin{bmatrix} 1 & -2 & 4 \\ -2 & 6 & 3 \\ 5 & 7 & 2 \end{bmatrix}$, find

(i) the cofactor of element in the second row first column and third row third column. **[1]**

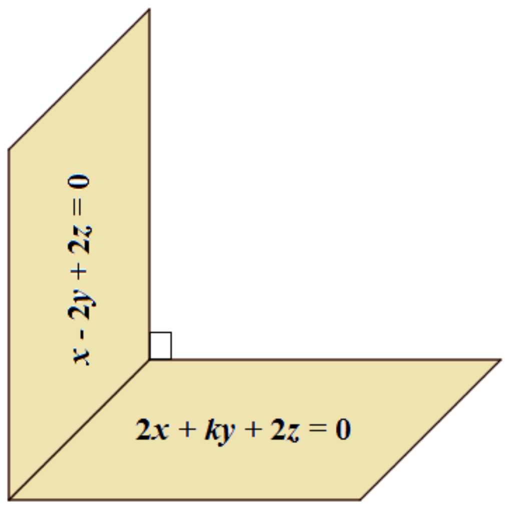
(ii) $adj\left(B^T\right)$. **[2]**

Question 5

a) Prove that $2\cos^{-1}x = \cos^{-1}\left(2x^2 - 1\right)$. **[3]**

b) Find the value of *k* for the two given planes. Using the value of *k*, find the equation of the plane passing through the origin and perpendicular to the two given planes.

[4]



Question 6

a) There are six boxes numbered 1, 2, …, 6. Each box needs to be filled up either with a red or a blue ball in such a way that at least four boxes contain blue balls and the boxes containing the blue balls are numbered consecutively. In how many ways can this be done. **[3]**

b) Find the integral of $a^{2x}x^2$. **[4]**

Question 7

a) Find the value of $P, P \neq 0$ if the points $(4, 2, 4)$, $(10, 2, -2)$ and $(2, 0, P)$ are the vertices of equilateral triangle. **[3]**

b) A school has planned to make a running track of 528 m enclosing a football field, the shape of which is rectangle with a semi-circle at each end for the conduct of track events. If the area of the rectangular portion is to be maximum, what would be the dimensions of the rectangle? $\left(\text{Use } \pi = \frac{22}{7}\right)$ **[4]**

Question 8

a) Solve: $\cos^{-1}\frac{4}{5}+\tan^{-1}x=\sin^{-1}\frac{17}{5\sqrt{13}}$ **[3]**

b) Show that $x^2+6xy+9y^2+4x+12y-5=0$ represents a pair of parallel straight lines and then find the perpendicular distance between the lines. **[4]**

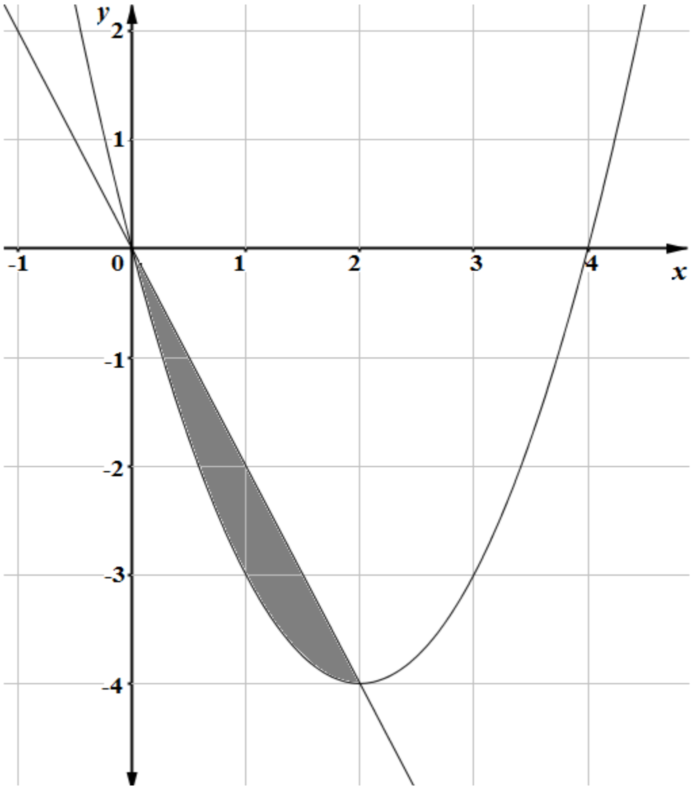
Question 9

a) If $y = a^{\tan x + a^{\tan x + a^{\tan x + \dots \infty}}}$, prove that $\frac{dy}{dx} = \frac{y(\sec^2 x \log a)}{1 - y \log a}$. **[3]**

b) Three volleyball players A, B and C are practicing during recess and are standing at these points (–1, 3, 2), (2, 3, 5) and (3, 5, –2) respectively. At what angles should they pass the ball to each other? **[4]**

Question 10

a) Find the area of the shaded region. **[3]**



b) If $A = \begin{bmatrix} -4 & 2 & -9 \\ 3 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix}$, find A^{-1} and use it in solving the system of linear equations: **[4]**

$$-4x_1 + 2x_2 - 9x_3 = 2$$
$$3x_1 + 4x_2 + x_3 = 5$$
$$x_1 - 3x_2 + 2x_3 = 8$$

Question 11

a) 65 teachers went for a trip to Gasa hot spring bath. 30 teachers went in one of the buses and the remaining in the other bus. What is the probability that
   (i) 2 particular teachers travelled in the same bus?
   (ii) 2 particular teachers were not in the same bus? **[3]**

b) Find the coordinates of vertex, focus, equation of directrix and length of latus rectum of the conic section $x^2 - 4x - 8y + 4 = 0$. **[4]**

Question 12

a) Following table represents the daily study hours of 7 students and their scores in the unit test. Calculate the rank correlation coefficient. What is the relationship between them? **[3]**

Hours	3	2.5	1	2	3	4	3.5
Scores	7	7	2	6	4	8	5

b) Solve: $x\dfrac{dy}{dx}-4y=x^6e^x$ **[4]**

Question 13

a) Find the value of *n* if ${}^{n}P_{2} = {}^{4}C_{2}$. **[3]**

b) If $x = \sin\theta$ and $y = \cos^3\theta$, prove that $\cos\theta \dfrac{d^2y}{dx^2} - \dfrac{y}{\cos\theta} = 3\sin^2\theta - 4\cos^2\theta$. **[4]**

FORMULAE

Strand A : Numbers and Operations

$a^2 - b^2 = (a+b)(a-b)$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

In QE $ax^2 + bx + c = 0, x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

${}^nP_r = \dfrac{n!}{(n-r)!}$

${}^nC_r = \dfrac{n!}{r!(n-r)!}$

$C_{ij} = (-1)^{i+j} M_{ij}$

$A^{-1} = \dfrac{1}{|A|} adjA$

$r = \sqrt{a^2 + b^2}$

$\tan\theta = \dfrac{b}{a}, \Rightarrow \theta = \tan^{-1}\left|\dfrac{b}{a}\right|$

$z = r(\cos\theta + i\sin\theta)$

$1 + 2 + 3 + ... + (n-1) = \dfrac{1}{2} n(n-1)$

$1^2 + 2^2 + 3^2 + ... + (n-1)^2 = \dfrac{1}{6} n(n-1)(2n-1)$

$1^3 + 2^3 + 3^3 + ... + (n-1)^3 = \left\{\dfrac{n(n-1)}{2}\right\}^2$

$\int_a^b f(x)dx = \lim_{h\to 0} h\left[\sum_{r=0}^{n-1} f(a+rh)\right]$

$A = \int_a^b y\,dx,\ V = \pi\int_a^b y^2\,dx$

$Volume\ of\ Cone = \dfrac{1}{3}\pi r^2 h$

$Volume\ of\ Sphere = \dfrac{4}{3}\pi r^3$

$Volume\ of\ Cylinder = \pi r^2 h$

$Volume\ of\ Prism = lbh$

$S.Area\ of\ Cone = \pi rl + \pi r^2$

$S.Area\ of\ Sphere = 4\pi r^2$

$S.Area\ of\ Cylinder = 2\pi rh + 2\pi r^2$

$S.Area\ of\ \text{Pr}ism = 2(lb + lh)$

$Area\ of\ sector = \dfrac{1}{2} r^2\theta$

Strand B : Patterns and Algebra

$y = x^n,\ y' = nx^{n-1}$

$y = cf(x),\ y' = cf'(x)$

$If\ \ y = uv,\ \dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$If\ \ y = \dfrac{u}{v} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$\int x^n dx = \dfrac{x^{n+1}}{n+1} + c$

$\int (ax+b)^n dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + c$

$\int uv\,dx = u\int v dx - \int\left(\dfrac{du}{dx}\int v dx\right)dx.$

$\dfrac{dy}{dx} + py = Q,\ \ I.F = e^{\int p dx},$

$General\ solution:$

$y(I.F) = \int Q(I.F.)dx + c$

Strand C : Measurement

$\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right),$

if $x, y \geq 0$ and $x^2 + y^2 \leq 1$

$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right),$

if $x, y > 0$ and $x^2 + y^2 \leq 1$

$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$, if $xy < 1$

$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right)$, if $xy > -1$

$2\tan^{-1} x = \tan^{-1}\dfrac{2x}{1-x^2}$

$= \sin^{-1}\dfrac{2x}{1+x^2} = \cos^{-1}\dfrac{1-x^2}{1+x^2}$

$$\cos ec^{-1}x = \sin^{-1}\frac{1}{x}$$
$$\sec^{-1}x = \cos^{-1}\frac{1}{x}$$
$$\cot^{-1}x = \tan^{-1}\frac{1}{x}$$

Strand D : Geometry

Angle between two lines
$$\cos\theta = \pm\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}\sqrt{a_2{}^2 + b_2{}^2 + c_2{}^2}}$$

If $a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$
$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

$$l = \pm\frac{a}{\sqrt{a^2 + b^2 + c^2}}$$
$$m = \pm\frac{b}{\sqrt{a^2 + b^2 + c^2}}$$
$$n = \pm\frac{c}{\sqrt{a^2 + b^2 + c^2}}$$
$$SP = ePM$$
$$\Rightarrow \sqrt{(x-\alpha)^2 + (y-\beta)^2} = \left|\frac{ax + by + c}{\sqrt{a^2 + b^2}}\right|$$
$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$
$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

Equation to the bisectors of angles :
$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

The point of intersection :
$$\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$$

Strand E: Data Management and Probability

$$r = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$
$$r = \frac{\sum(x - \bar{x}) - \sum(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2\sum(y - \bar{y})^2}}$$
$$r = 1 - \frac{6\sum D^2}{n\left(n^2 - 1\right)},$$
$$Correction\ factor = \frac{1}{12}\left(m^3 - m\right)$$
$$r = \pm\sqrt{b_{yx} \times b_{xy}}$$
$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - \left(\sum x\right)^2}$$
$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$$
$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}\left(x - \bar{x}\right)$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A) + P(\bar{A}) = 1$$
$$P(B/A) = \frac{P(A \cap B)}{P(A)}, P(A) \neq 0$$
$$P(A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

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