SECTION A [30 MARKS]
ANSWER ALL QUESTIONS

Question 1 [30]

Direction: For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than ONE alternative. If there are more than ONE choice circled, NO score will be awarded.
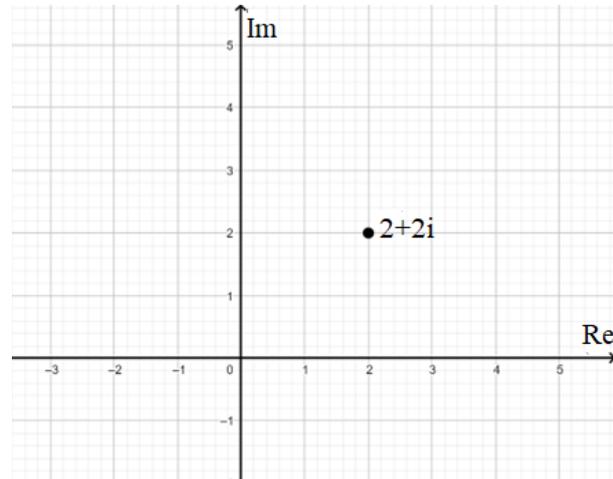
i)   The polar form of complex number for the point shown in the graph is

A $-2\sqrt{2}\, Cis\, \dfrac{\pi}{4}.$

B $-2\sqrt{2}\, Cis\, \dfrac{\pi}{3}.$

C $2\sqrt{2}\, Cis\, \dfrac{\pi}{4}.$

D $2\sqrt{2}\, Cis\, \dfrac{\pi}{3}.$

ii)   Equation of a plane that passes through the points $(-2, 0, 0), (0, 1, 0)$ and $(0, 0, -1)$ is

A $x + 2y + 2z - 2 = 0.$

B $x - 2y - 2z - 2 = 0.$

C $x - 2y + 2z + 2 = 0.$

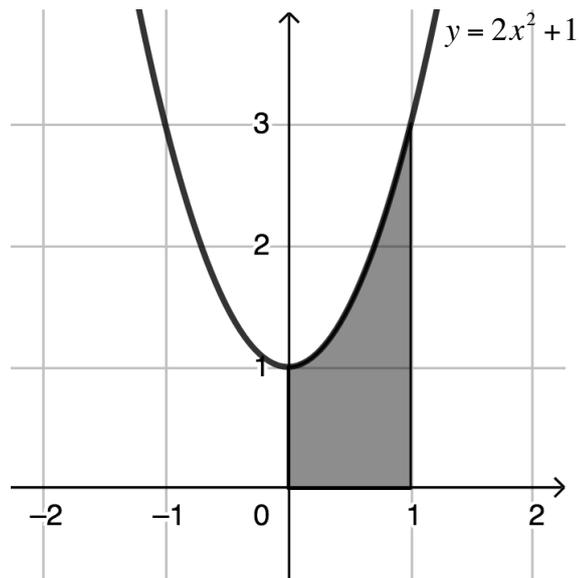D $x + 2y - 2z - 2 = 0.$

iii)   What is the area of the shaded region?

A $\dfrac{1}{3}$ sq. units

B $\dfrac{2}{3}$ sq. units

C $\dfrac{4}{3}$ sq. units

D $\dfrac{5}{3}$ sq. units

iv) What is the degree and order of the differential equation $\dfrac{dy}{dx} - \dfrac{2}{dy/dx} = y$?

     A   Order: 1, degree: 1
     B   Order: 1, degree: 2
     C   Order: 2, degree: 1
     D   Order: 2, degree: 2

v) The principal value of $\tan^{-1}\left(\cot\dfrac{5\pi}{4}\right)$ is

     A $\dfrac{\pi}{4}$.

     B $\dfrac{5\pi}{4}$.

     C $\dfrac{3\pi}{4}$.

     D $\dfrac{\pi}{2}$.

vi) If $A = \begin{pmatrix} 3 & -1 \\ -1 & 0 \end{pmatrix}$ and $f(x) = x^2 - 3x + 1$. Find $f(A)$.

     A $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

     B $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

     C $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

   D $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

vii) There are 10 lamps in a hall. Each lamp can be switched on independently. The number of ways in which the hall can be illuminated is

     A   1024.
     B   1023.
     C   100.
     D   10!.

viii) Find the value of P if the equation for the pair of straight lines is
$x^2 + 2Pxy - 6y^2 + 7x + 31y - 18 = 0$ and angle between the lines is 45^0.

    A $\dfrac{1}{2}$

    B $\dfrac{3}{2}$

    C $\dfrac{1}{4}$

    D $\dfrac{3}{4}$

ix) $\int_0^{\frac{\pi}{2}} \cos|x|\, dx$ is

    A   0.
    B   1.
    C   2.
    D   3.

x)   The gradient of a function, $\log(\sin x)$ at $x = \dfrac{\pi}{3}$ is

    A $-\dfrac{1}{\sqrt{3}}.$

    B $-\sqrt{3}.$

    C $\dfrac{1}{\sqrt{3}}.$

    D $\sqrt{3}.$

xi)   Distance travelled by an ant moving in a straight line at time t is described by
$s = 8t^3 - 6t^4$. What is the maximum acceleration with which the ant is moving?

    A   24
    B   20
    C   12
    D   8

xii)   Which of the points are collinear?

    A   A(0, 2, 4), B(4, 3, 1) and C(2, 1, 2)
    B   A(1, 2, 3), B(3, 8, 1) and C(7, 20, − 3)
    C   A(1, 4, 2), B(−2, 1, 2) and C(2, − 3, 4)
    D   A(4, 2, 4), B(10, 2, − 2) and C(2, 0, − 4)

xiii)   Find the equation for the given conic section whose eccentricity is $\dfrac{1}{\sqrt{2}}$.

A $\dfrac{(x-1)^2}{36}+\dfrac{(y-2)^2}{18}=1$

B $\dfrac{(x-1)^2}{18}+\dfrac{(y-2)^2}{36}=1$

C $\dfrac{(x-2)^2}{36}+\dfrac{(y-1)^2}{18}=1$

D $\dfrac{(x-2)^2}{18}+\dfrac{(y-1)^2}{36}=1$
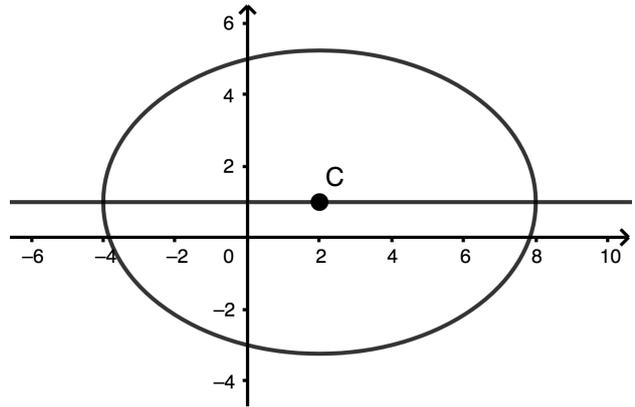
xiv)   The correlation between x and y is $\dfrac{2}{5}$ and the sum of the squares of the differences in ranks is 12. How many pairs were taken to calculate the correlation?

A   5
B   6
C   8
D   10

xv)   There are two bags. First bag contains 3 red, 4 yellow and 2 blue balls and the other bag contains 3 red, 3 yellow and 3 blue balls. A die is rolled. If a multiple of 3 is rolled, a ball is drawn from the first bag, otherwise a ball is drawn from the second bag. What is the probability of drawing a yellow ball?

A $\dfrac{20}{81}$

B $\dfrac{19}{54}$

C $\dfrac{10}{27}$

D $\dfrac{7}{18}$

SECTION B [70 MARKS]
ATTEMPT ANY TEN QUESTIONS

Question 2

a)   If $\sqrt{\dfrac{y}{x}} + \sqrt{\dfrac{x}{y}} = 6$, show that $\dfrac{dy}{dx} = \dfrac{x - 17y}{17x - y}$.

[4]

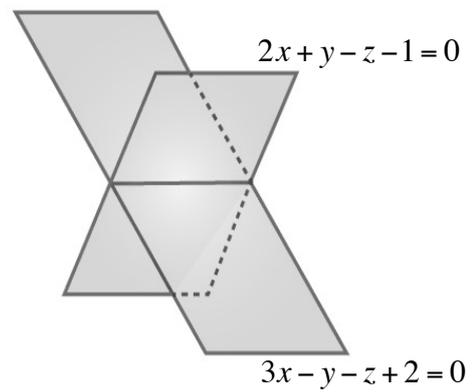b)   Prove the theorem $adj(AB) = (adj\ B)(adj\ A)$ with an example.   **[3]**

Question 3

a)   Find the equation of the plane through the intersection of given planes and passing   **[4]**
     through the point (2, 1, -1).

$$2x + y - z - 1 = 0$$

$$3x - y - z + 2 = 0$$

b) Solve: $\cos\left(\sin^{-1}\dfrac{x}{\sqrt{1+x^2}}\right) = \sin\left(\cot^{-1}\dfrac{3}{4}\right)$ **[3]**

Question 4

a) Check and prove with an example whether the given statement **[3]**

$\int\dfrac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$ is true or false.

b) Calculate the line of best fit for the following data and then estimate the value of height when the weight is 56 kg. **[4]**

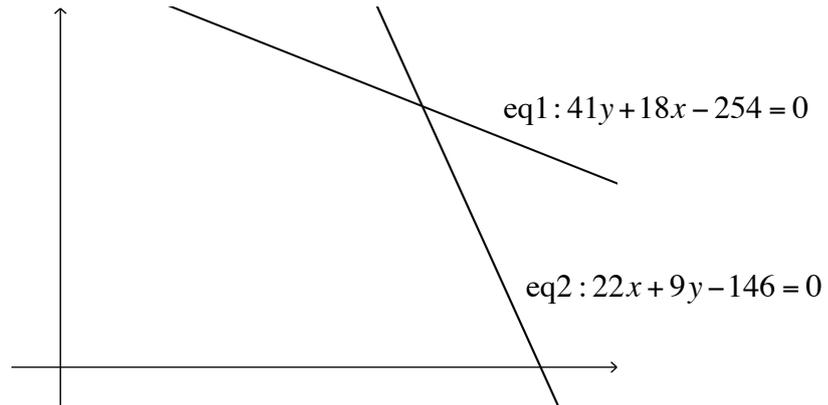| Height(cm)	156	154	150	172	150	168	170	168
Weight(kg)	60	64	61	67	55	52	70	67

Question 5

a) Find the value of P and Q, if the following equation represents a pair of **[4]** perpendicular lines $6x^2 + 5xy - Py^2 + 7x + Qy - 5 = 0$.

b) The given equations eq1 and eq2 are regression equations of X on Y and Y on X respectively. Is the statement true? Compute b_{xy}, b_{yx}, $\bar{x}$ and $\bar{y}$. **[3]**

$$eq1 : 41y + 18x - 254 = 0$$

$$eq2 : 22x + 9y - 146 = 0$$

Question 6

a) A school literary committee has organized 3 literary activities to observe the literary week. For each activity there is only one prize. If the same 10 students have participated in all three activities, how can the prize be distributed if, **[3]**
   i)   a student can receive any number of prizes?
   ii)   a student cannot receive all the three prizes?

b) Find the locus of a point such that the difference of its distances from $(6, -2)$ and $(-4, -2)$ is always equal to 8 and then identify the locus of the point. **[4]**

Question 7

a)   Solve: $\cos^{-1}\left(\dfrac{dy}{dx}\right) = x + y$

[3]

b) The ages of 7 married couples were collected to find the correlation between the ages **[4]** of husbands(x) and wives (y) and the following results were listed:

$$\sum x = 263, \sum y = 221, \sum xy = 8929, \sum x^2 = 10591 \ and \ \sum y^2 = 7647.$$

On further inspection, the pair $(x = 35, y = 23)$ was recorded incorrectly. The correct pair was $(x = 38, y = 33)$. What might be the correct correlation between the ages of husbands and wives? Interpret the result. Provide the possible list of ages of 7 wives that sum up to 231.

Question 8

a) Evaluate: $\int_{-\pi}^{\pi} \sin^3 x \, dx$ **[2]**

b) Match the following and write your answer in the table given below. **[2]**

Function	Range
1. $\sin^{-1} x$	a. $[0, \pi]$
2. $\cos^{-1} x$	b. $[0, \pi] - \left\{ \dfrac{\pi}{2} \right\}$
3. $\sec^{-1} x$	c. $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2} \right] - \{0\}$
4. $\csc^{-1} x$	d. $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2} \right]$

Function	Range
1. $\sin^{-1} x$	
2. $\cos^{-1} x$	
3. $\sec^{-1} x$	
4. $\csc^{-1} x$	

c) A mother, a father and a child line up at random for a family picture. What is the probability of the child being on one end if the father is in the middle? **[3]**
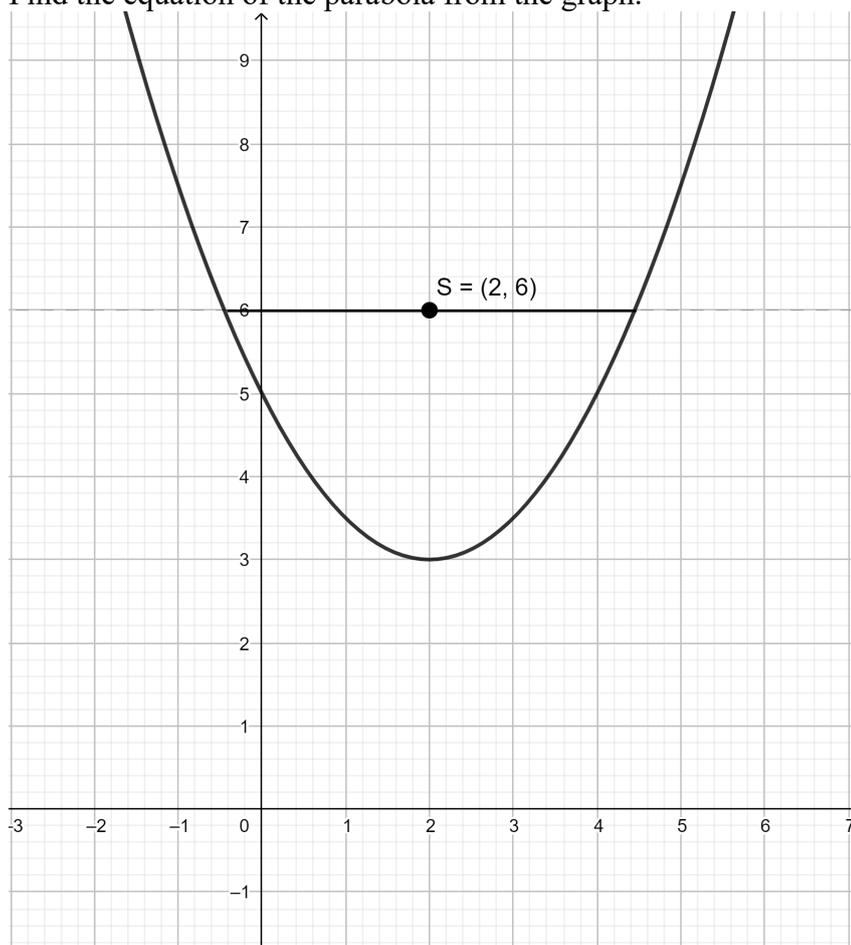
Question 9

a) Find the equation of the parabola from the graph. **[3]**

S = (2, 6)

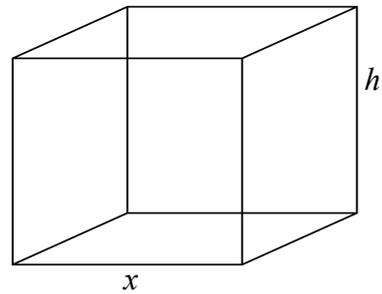b) Evaluate: $\int \dfrac{\sin x \cos x}{\cos^2 x(\sin x + 1)}\,dx$

[4]

Question 10

[3]

a) A man decide to construct an open tank with a square base to hold P m^3 of water for his new house. Show that the cost of the material will be least if $h = \dfrac{1}{2}x$.
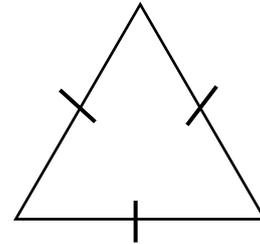
b)   Find the equation of the lines parallel to the lines $x^2 + xy - 6y^2 + 7x + 31y - 18 = 0$ **[4]**
and passing through the point $(-1, 2)$.

Question 11

a) A religious body is planning to construct a unique religious structure as shown in the diagram. Find the value of P, if coordinates of the vertices are $(4,2,4),(10,P,-2)$ and $(2,0,-4)$ **[3]**
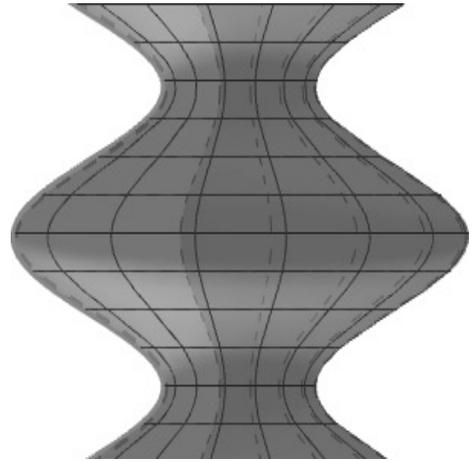
b) Find the number of (i) combinations (ii) permutations of four letters taken from the word CERTIFICATE? **[4]**

Question 12

a)   A company wants to create a vessel of a given shape which is formed by revolving   **[3]**
     the graph of $y = \sin 2x + 2$ about the x - axis over the interval $0 \le x \le \pi$. What is the
     volume of the vessel?

b) A boy throws a stone upward and the velocity after t seconds is **[4]**
$v(t) = at^2 - bt + c, 0 \leq t \leq 8,$ where a, b and c are constants. It was found that at time $t = 1, 2,$ and 4 seconds , the velocity of the stone was 2, 3 and 11 m/s respectively. Find the speed at 6 seconds using matrix method.

Question 13

a) If $z = x + yi$, $\omega = \dfrac{1 + zi}{2z + 3}$ and $|\omega| = 1$, find the locus of z and illustrate it on a complex plane.

[3]

b) Prove that $x\dfrac{d^2y}{dx^2} + (1-4y)\dfrac{dy}{dx} = 0$ if $y = \tan\left(\log x^2\right)$.

[4]

FORMULAE

Strand A : Numbers and Operations

$a^2 - b^2 = (a+b)(a-b)$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

In QE $ax^2 + bx + c = 0, x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$^nP_r = \dfrac{n!}{(n-r)!}$

$^nC_r = \dfrac{n!}{r!(n-r)!}$

$C_{ij} = (-1)^{i+j} M_{ij}$

$A^{-1} = \dfrac{1}{|A|} adjA$

$r = \sqrt{a^2 + b^2}$

$\tan\theta = \dfrac{b}{a}, \Rightarrow \theta = \tan^{-1}\left|\dfrac{b}{a}\right|$

$z = r(\cos\theta + i\sin\theta)$

Strand B : Patterns and Algebra

$y = x^n, \; y' = nx^{n-1}$

$y = cf(x), \; y' = cf'(x)$

If $y = uv, \dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

If $y = \dfrac{u}{v} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$\displaystyle\int x^n dx = \dfrac{x^{n+1}}{n+1} + c$

$\displaystyle\int (ax+b)^n dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + c$

$\displaystyle\int uv\, dx = u\int v dx - \int\left(\dfrac{du}{dx}\int v dx\right) dx.$

$\dfrac{dy}{dx} + py = Q, \; I.F = e^{\int pdx},$

General solution :

$y(I.F) = \displaystyle\int Q(I.F.)\, dx + c$

$1 + 2 + 3 + \ldots + (n-1) = \dfrac{1}{2}n(n-1)$

$1^2 + 2^2 + 3^2 + \ldots + (n-1)^2 = \dfrac{1}{6}n(n-1)(2n-1)$

$1^3 + 2^3 + 3^3 + \ldots + (n-1)^3 = \left\{\dfrac{n(n-1)}{2}\right\}^2$

$\displaystyle\int_a^b f(x)dx = \lim_{h\to 0} h\left[\sum_{r=0}^{n-1} f(a+rh)\right]$

$A = \displaystyle\int_a^b y\, dx, \; V = \pi\int_a^b y^2\, dx$

$Volume\, of\, Cone = \dfrac{1}{3}\pi r^2 h$

$Volume\, of\, Sphere = \dfrac{4}{3}\pi r^3$

$Volume\, of\, Cylinder = \pi r^2 h$

$Volume\, of\, Prism = lbh$

$S.Area\, of\, Cone = \pi rl + \pi r^2$

$S.Area\, of\, Sphere = 4\pi r^2$

$S.Area\, of\, Cylinder = 2\pi rh + 2\pi r^2$

$S.Area\, of\, Prism = 2(lb + lh)$

$Area\, of\, sector = \dfrac{1}{2}r^2\theta$

Strand C : Measurement

$\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right),$
$\qquad\qquad if\, x,y \geq 0 \; and \; x^2 + y^2 \leq 1$

$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right),$
$\qquad\qquad if\, x,y > 0 \; and \; x^2 + y^2 \leq 1$

$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right), if\, xy < 1$

$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right), if\, xy > -1$

$2\tan^{-1} x = \tan^{-1}\dfrac{2x}{1-x^2}$
$\qquad = \sin^{-1}\dfrac{2x}{1+x^2} = \cos^{-1}\dfrac{1-x^2}{1+x^2}$

$\cos ec^{-1} x = \sin^{-1}\dfrac{1}{x}$

$\sec^{-1} x = \cos^{-1}\dfrac{1}{x}$

$\cot^{-1} x = \tan^{-1}\dfrac{1}{x}$

Strand D : Geometry

Angle between two lines

$$\cos\theta = \pm\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

If $a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

$$l = \pm\frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \pm\frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \pm\frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$$SP = ePM$$

$$\Rightarrow \sqrt{(x-\alpha)^2 + (y-\beta)^2} = \left|\frac{ax + by + c}{\sqrt{a^2 + b^2}}\right|$$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

Equation to the bisectors of angles :

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

The point of intersection :

$$\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$$

Strand E: Data Management and Probability

$$r = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

$$r = \frac{\sum(x - \bar{x}) - \sum(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2 \sum(y - \bar{y})^2}}$$

$$r = 1 - \frac{6\sum D^2}{n(n^2 - 1)},$$

$$\text{Correction factor} = \frac{1}{12}(m^3 - m)$$

$$r = \pm\sqrt{b_{yx} \times b_{xy}}$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - (\sum x)^2}$$

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A) + P(\bar{A}) = 1$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}, P(A) \neq 0$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

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