SECTION A [30 MARKS]

ANSWER **ALL** THE QUESTIONS

Question 1 **[30]**

For each question there are FOUR responses: A, B, C and D. Choose the corresponding letter of your response and CIRCLE it neatly. NO score will be awarded if you circle more than ONE letter.

i. What is the value of $\frac{1+\omega}{\omega+\omega^2}$, if ω is cube root of unity?

A 0
B 1
C ω
D ω^2

ii. A school gardener wants to plant 10 different varieties of flowers around a circular walkway. How many different ways can the flowers be planted?

A 3628800
B 1814400
C 362880
D 181440

iii. If A is a $3\times r$ matrix, B is a $5\times s$ matrix and the product matrix $AB=\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ then the value of r and s are

A $r=2$ and $s=5$.
B $r=5$ and $s=2$.
C $r=3$ and $s=2$.
D $r=2$ and $s=3$.

iv. If $\frac{d}{dx}(\log_e x) = \frac{1}{x}$, then $\frac{d}{dx}(\log_2 x)$ is

A $\frac{1}{x \log_e 2}$.

B $\frac{1}{x \log_2 e}$.

C $\frac{\log_e 2}{x}$.

D $\frac{\log_2 e}{x}$.

v. Which of the following functions satisfies the property $\int_{-a}^{a} f(x)dx = 0$?

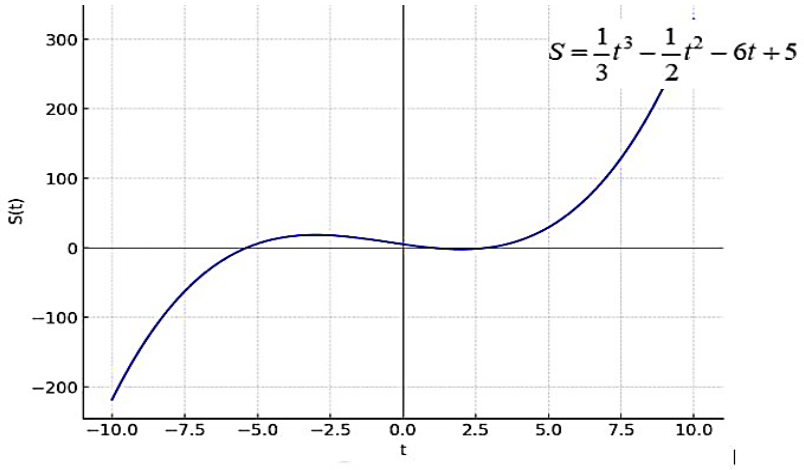
A $|\sin x|$

B $x^6 + 1$

C $x^2 \sin x$

D $(x^2 + 1)\cos x$

vi. The graph represents the distance travelled by a particle in time t seconds. Study the graph and calculate the acceleration after 3 seconds.



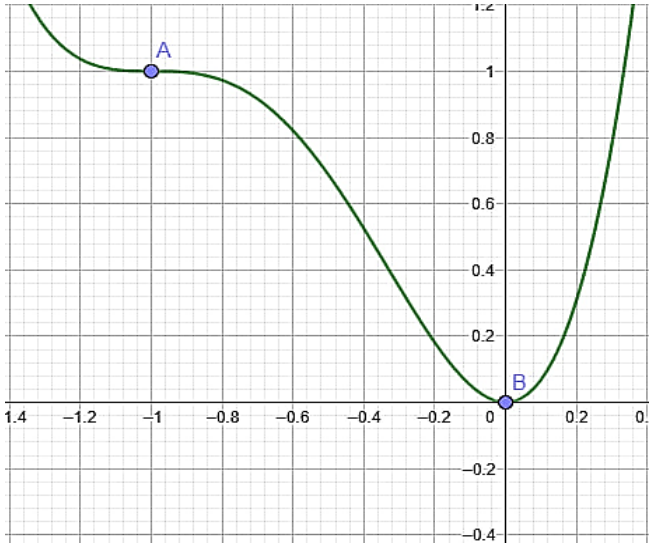
A $6.0\, ms^{-2}$

B $5.0\, ms^{-2}$

C $2.5\, ms^{-2}$

D $0.0\, ms^{-2}$

vii. Which of the following is **NOT** true about points A and B in the given diagram.



A $\frac{dy}{dx} = 0$ at both A and B

B $\frac{d^2y}{dx^2} > 0$ at B

C $\frac{d^3y}{dx^3} \neq 0$ at A

D $\frac{d^2y}{dx^2} \neq 0$ at A

viii. Students are arguing about the order and degree of the differential equation $y = x\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dy}\right)$. The arguments are:

Pema: order 1 and degree 2
Dema: order 1 and degree 3
Kumar: order 2 and degree 2
Gyem: order 2 and degree 3

Whose argument is correct?

A Pema
B Dema
C Kumar
D Gyem

ix. Evaluate: $\sin\left[\cos^{-1}\left(\frac{2}{3}\right)+\sin^{-1}\left(\frac{2}{3}\right)\right]$

A $\frac{\pi}{2}$

B $\frac{2}{3}$

C 1

D 0

x. Karma and Bikash are participating in a national shotput championship. The throwing mark is at point $A(1,3,7)$. Karma threw the shotput and it landed at point $B(6,3,2)$. If the distance thrown by Bikash is two-third of the distance thrown by Karma, where will Bikash's shotput land?

A (3, 3, 5)

B $(11, 3, -3)$

C $\left(\frac{7}{2}, 3, \frac{9}{2}\right)$

D $\left(\frac{13}{3}, 3, \frac{11}{3}\right)$

xi. Find the intercepts of the plane $6x-3y+2z-6=0$ on the axes.

A 1, 2, 3

B 6, 3, 2

C $1, -2, 3$

D $6, -3, 2$

xii. In the ongoing BoB football Premier league, the probability of winning the title by Paro FC is $\frac{3}{4}$ and Thimphu City is $\frac{1}{4}$. Which of the following events best describes the above situation?

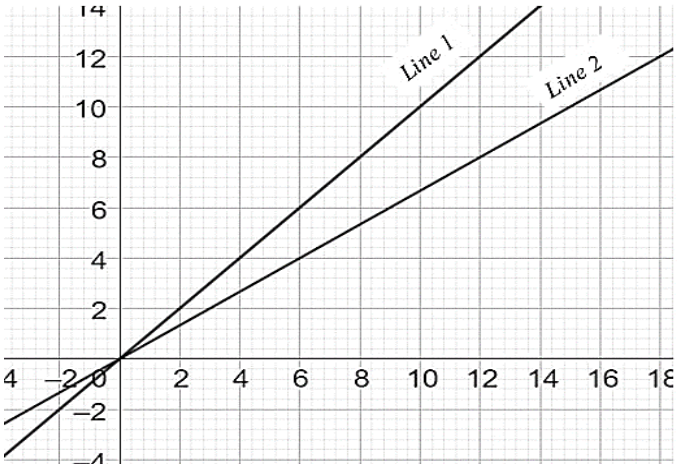
A Mutually exclusive, independent and exhaustive

B Mutually exclusive, dependent and exhaustive

C Mutually exclusive, dependent and not exhaustive

D Not mutually exclusive, independent and exhaustive

xiii. Combined equation of lines given in the graph is represented by $2x^2 - 5xy + 3y^2 = 0$. What are the equations of line 1 and line 2?



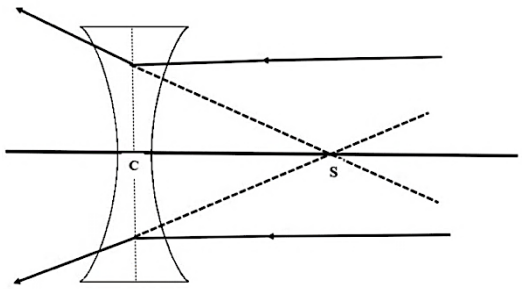
A Line 1: $x - y = 0$
Line 2: $2x - 3y = 0$

B Line 1: $2x - 3y = 0$
Line 2: $x - y = 0$

C Line 1: $2x + 3y = 0$
Line 2: $x - y = 0$

D Line 1: $x - y = 0$
Line 2: $2x + 3y = 0$

xiv. The given biconcave lens can be modeled by the equation $16x^2 - 9y^2 = 576$. Find the coordinate of point 'S'.



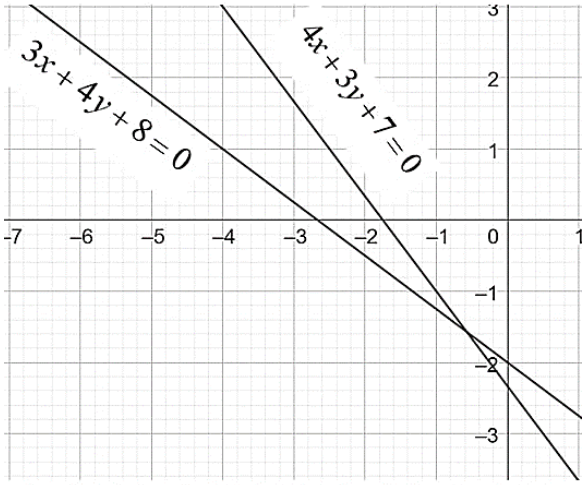
A $(0, -10)$

B $(-10, 0)$

C $(0, 10)$

D $(10, 0)$

xv. Find the correlation coefficient using the given regression lines.



A 0.75

B 0.25

C –0.25

D –0.75

SECTION B [70 MARKS]

ANSWER **ANY TEN** QUESTIONS

Question 2

a) Find the equation of plane passing through the points $(1,-1,2)$ and $(-3,2,-2)$, and perpendicular to plane $x+2y-3z+7=0$. **[4]**

b) B-mobile numbers have two series: 17XXXXXX and 16XXXXXX. If the third digit can be any number from 5 to 9, how many unique mobile numbers can be generated? **[3]**

Question 3

a) The ranks of 5 students in 3 subjects are given in the table. Use rank correlation to determine the best subject combination with Mathematics. **[4]**

Students	Mathematics	Chemistry	Physics
A	2	5	2
B	4	1	3
C	5	2	5
D	1	3	4
E	3	4	1

b) Find $\frac{dy}{dx}$ if $y = \cos^{-1}\left(1-2x^2\right)$. **[3]**

Question 4

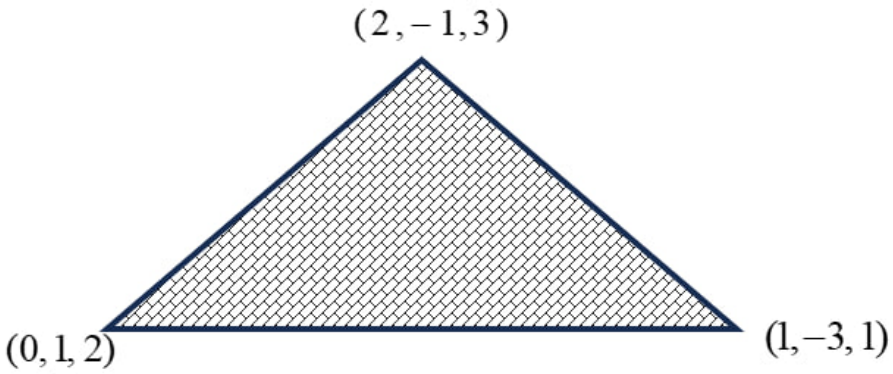
a) Evaluate: $\int \frac{2x+3}{(x-1)(x^2+1)}\,dx$ **[4]**

b) Solve: $\tan\left(\cos^{-1} x\right) = \sin\left[\cot^{-1}\left(\frac{1}{2}\right)\right]$ **[3]**

Question 5

a) Kinley invests in three types of stock markets: RICB, BNBL and STCB. He invests a total of Nu 50,000. The expected annual returns are 8% for RICB, 6% for BNBL, and 10% for STCB, yielding a total return of Nu 4,200 annually. If his investment in RICB is 3 times the investment in BNBL, use matrix to determine the stock which yields maximum dividend? **[4]**

b) Find the cost of painting the right triangular roof if the cost of painting 1 square unit is Nu 120. **[3]**



Question 6

a) A car travels along the straight road. Its distance d in kilometers at time t in hours is given by the equation $d = 40t + 2t^2$. The fuel consumption rate c in litres per hour is given by $c = 0.1t^2 + 0.8t + 2$.

i. Calculate the speed of the car at $t = 3$ hours. **[1.5]**

ii. What is the rate of change in distance with respect to fuel consumption in 2 hours? **[2.5]**

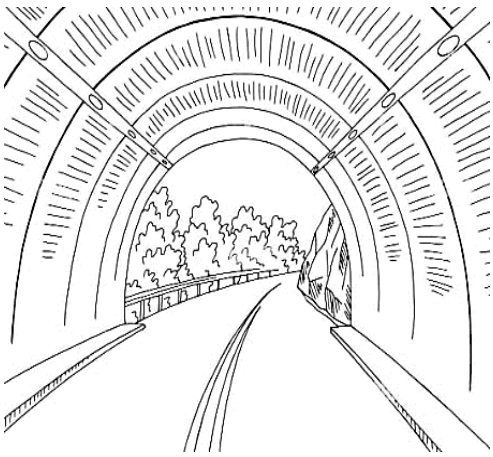
b) Match the following: **[3]**

Column A		Column B
i)	Independent event	a) $P(A \cup B) = P(A) + P(B) + P(A \cap B)$
ii)	Complementary event	b) $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$
iii)	Conditional event	c) $P(A \cap B) = P(A) \times P(B)$
		d) $P(A) + P(B) = 1$
		e) $P\left(\overline{A}\right) = 1 - P\left(A\right)$

Column A		Column B
i)	Independent event	
ii)	Complementary event	
iii)	Conditional event	

Question 7

a) The given tunnel can be modeled by an equation $25x^2 + 16y^2 = 400$. Decide a dimension of vehicle that can safely pass through this tunnel assuming that the vehicle drives through the centre of tunnel road. **[4]**



b) Show that $\beta(Adj\beta)+(Adj\beta)\beta=2(|\beta|I)$ if $\beta=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. **[3]**

Question 8

a) Find the limits of the integration for the curve $4x^2 + 5y^2 = 100$ and the volume of the solid formed by one complete rotation about x-axis. **[4]**

b) Find the distance between the parallel planes **[3]**
$4x+3y-12z-4=0$ and $4x+3y-12z+6=0$.

Question 9

a) Find the general solution of $\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$. **[4]**

b) Express the complex number $z = 5\sqrt{3} - 5i$ into polar form. **[3]**

Question 10

a) Check whether the equation $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of straight lines and find the equations. **[5]**

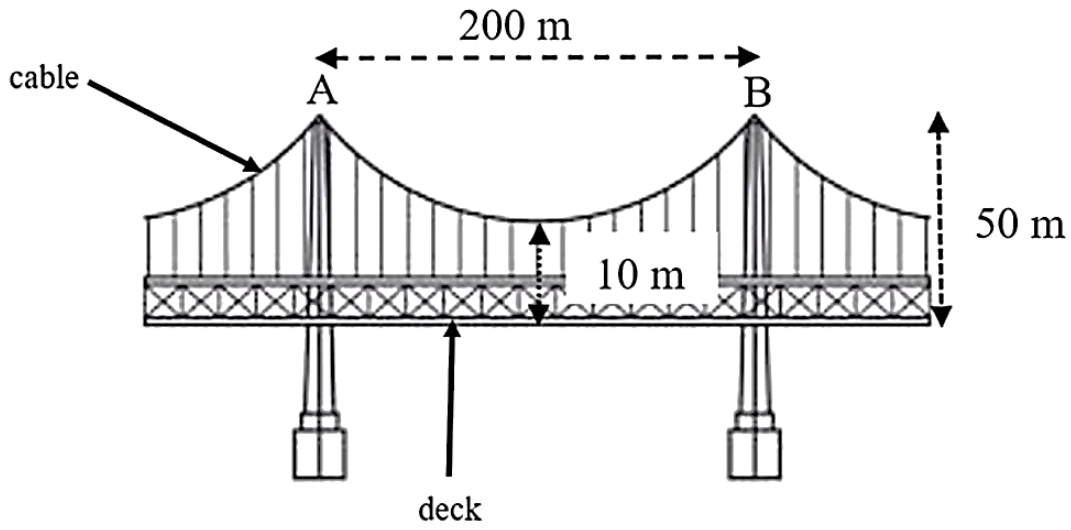
b) Examine the principal values of the following functions. **[2]**

Function	Value	Principal Value
i) $\sin^{-1} x$	$\frac{2\pi}{3}$	----------
ii) $\cos^{-1} x$	$\frac{2\pi}{3}$	----------
iii) $\tan^{-1} x$	$\frac{2\pi}{3}$	----------
iv) $\cot^{-1} x$	$\frac{2\pi}{3}$	----------

Question 11

a) An engineer is designing a square-based open tank to address water shortages. The tank is designed to hold V cubic units of water. If you are the engineer, decide the height of the tank to ensure cost-effectiveness. **[4]**

b) Find the equation of an arc formed by the suspension bridge cable between towers A and B, assuming a vertical axis through the center of the towers and a horizontal axis through the base of the deck. **[3]**



Question 12

a) Yethro and Damchoe appear for an interview for two vacancies. They will be selected if they score at least 60% in each criterion.

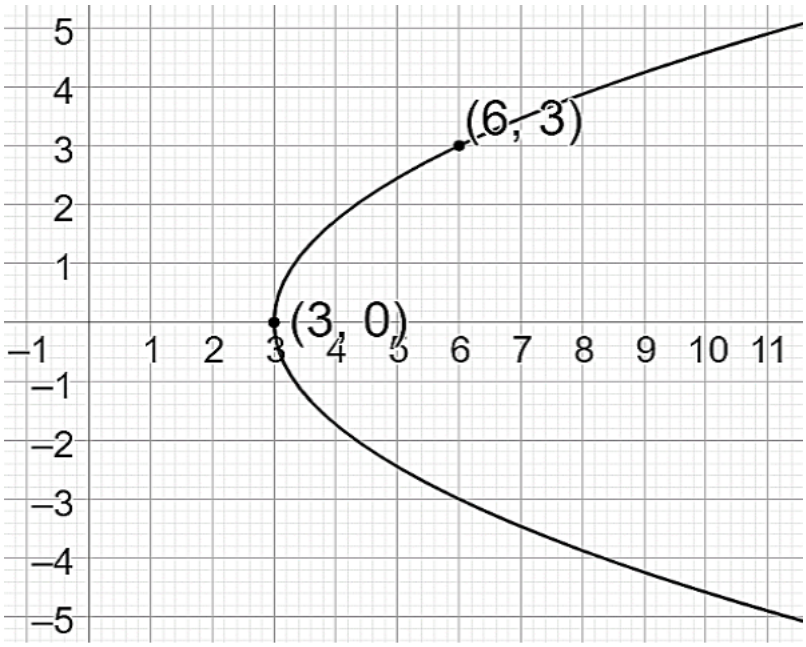
Criteria	**Viva [10]**	**Academic [10]**	**Leadership [5]**	**Volunteerism [5]**	**Presentation [5]**	**Total**
Yethro	4	5	4	5	3	21
Damchoe	5	6	2	2	1	16

What is the probability that

i. only one of them will be selected? **[3]**

ii. at least one of them will be selected? **[1]**

b) Find the area bounded by the curve, the x-axis and $x = 10$. **[3]**



Question 13

a) Illustrate and describe the region of Argand plane represented by the inequality **[4]**

$\left|\frac{z-3i}{z+3i}\right| \leq \sqrt{2}$.

b) A study is conducted to examine the relationship between the continuous assessment (CA) mark and the mark obtained in a Mathematics exam. The data collected from 5 students are as follows: **[3]**

Student	**A**	**B**	**C**	**D**	**E**
CA	5	10	3	8	6
Exam	60	75	50	70	65

If a student has scored 7 marks in CA, predict his score in the exam.

FORMULAE

Strand A: Numbers and Operations

$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

If $ax^2 + bx + c = 0,\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

${}^nP_r = \dfrac{n!}{(n-r)!}$

${}^nC_r = \dfrac{n!}{r!(n-r)!}$

$C_{ij} = (-1)^{i+j} M_{ij}$

$A^{-1} = \dfrac{1}{|A|} adj.A$

$r = \sqrt{a^2 + b^2}$

$\tan\theta = \dfrac{b}{a} \Rightarrow \theta = \tan^{-1}\left|\dfrac{b}{a}\right|$

$z = r\left(\cos\theta + i\sin\theta\right)$

Strand B: Patterns and Algebra

$y = x^n,\ y' = nx^{n-1}$

$y = cf(x),\ y' = cf'(x)$

If $y = uv,\ \dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

If $y = \dfrac{u}{v},\ \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$\int x^n dx = \dfrac{x^{n+1}}{n+1} + c$

$\int (ax+b)^n dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + c$

$\int uvdx = x\int vdx - \int \left\{\dfrac{du}{dx}\int vdx\right\}dx$

$\dfrac{dy}{dx} + Py = Q,\ I.F. = e^{\int Pdx}$

$y(I.F.) = \int Q(I.F.)dx + c$

$1 + 2 + 3 + ... + (n-1) = \dfrac{1}{2}n(n-1)$

$1^2 + 2^2 + ... + (n-1)^2 = \dfrac{1}{6}n(n-1)(2n-1)$

$1^3 + 2^3 + 3^3 + ... + (n-1)^3 = \left[\dfrac{n(n-1)}{2}\right]^2$

$\int_a^b f(x)dx = \underset{h\to 0}{\text{Lim}}\, h\left[\sum_{r=0}^{n-1} f(a+rh)\right]$

$A = \int_a^b ydx,\ V = \pi\int_a^b y^2 dx$

Volume of Cone: $\dfrac{1}{3}\pi r^2 h$

Volume of Sphere: $\dfrac{4}{3}\pi r^3$

Volume of Cylinder: $\pi r^2 h$

Volume of Prism: $Base\ Area \times height$

S. Area of Cone: $\pi rl + \pi r^2$

S. Area of Sphere: $4\pi r^2$

S. Area of Cylinder: $2\pi rh + \pi r^2$

Area of sector: $\dfrac{1}{2}r^2\theta$

Strand C: Measurement

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

$\sin 2A = 2\sin A\cos A;$

$\cos 2A$

$= \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1;$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A};$

$\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right),$

$If\ x, y \geq 0\ \&\ x^2 + y^2 \leq 1;$

$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{1-y^2}\cdot\sqrt{1-x^2}\right),$

$If\ x, y \geq 0\ \&\ x^2 + y^2 \leq 1.$

$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)\ if\ xy < 1;$

$2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right);$

$2\cos^{-1} x = \cos^{-1}\left(2x^2 - 1\right);$

$2\tan^{-1} x = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$

$= \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

$= \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right), \text{if } x^2 < 1$

$3\sin^{-1} x = \sin^{-1}\left(3x - 4x^3\right)$

$3\cos^{-1} x = \cos^{-1}\left(4x^3 - 3x\right)$

$3\tan^{-1} x = \tan^{-1}\left(\dfrac{3x - x^3}{1-3x^2}\right)$

$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) \text{ if } xy > -1;$

Strand D: Geometry

$\cos\theta = \pm\dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2}\sqrt{{a_2}^2 + {b_2}^2 + {c_2}^2}}$

$If\ a_1x + b_1y + c_1z = 0 \quad a_2x + b_2y + c_2z = 0,$

$\dfrac{x}{b_1c_2 - b_2c_1} = \dfrac{y}{c_1a_2 - c_2a_1} = \dfrac{z}{a_1b_2 - a_2b_1}$

$SP = ePM \Rightarrow \sqrt{(x-\alpha)^2 + (y-\beta)^2} = \left|\dfrac{ax + by + c}{\sqrt{a^2 + b^2}}\right|.$

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

Equation of angle bisectors: $\dfrac{x^2 - y^2}{a - b} = \dfrac{xy}{h}$

$l = \pm\dfrac{a}{\sqrt{a^2 + b^2 + c^2}}$

$m = \pm\dfrac{b}{\sqrt{a^2 + b^2 + c^2}}$

$n = \pm\dfrac{c}{\sqrt{a^2 + b^2 + c^2}}$

$\tan\theta = \left|\dfrac{2\sqrt{h^2 - ab}}{a + b}\right|$

Point of intersection:

$\left(\dfrac{hf - bg}{ab - h^2}, \quad \dfrac{gh - af}{ab - h^2}\right)$

Strand E: Data Management and Probability

$r = \dfrac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$

$r = \dfrac{\sum\left(x - \bar{x}\right) - \sum\left(y - \bar{y}\right)}{\sqrt{\sum\left(x - \bar{x}\right)^2 - \sum\left(y - \bar{y}\right)^2}}$

$r = 1 - \dfrac{6\left(\sum D^2 + \text{correction factors}\right)}{n(n^2 - 1)}$

Correction factor: $\dfrac{1}{12}\left(m^3 - m\right) + ...$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A) + P(\bar{A}) = 1.$

$r = \pm\sqrt{b_{yx}b_{xy}}$

$b_{yx} = r\dfrac{\sigma_y}{\sigma_x} = \dfrac{n\sum xy - \sum x\sum y}{n\sum x^2 - \left(\sum x\right)^2}$

$b_{xy} = r\dfrac{\sigma_x}{\sigma_y} = \dfrac{n\sum xy - \sum x\sum y}{n\sum y^2 - \left(\sum y\right)^2}$

$y - \bar{y} = b_{yx}\left(x - \bar{x}\right);\ x - \bar{x} = b_{xy}\left(y - \bar{y}\right)$

$P(B / A) = \dfrac{P(A \cap B)}{P(A)},\ P(A) \neq 0.$

$P(A / B) = \dfrac{P(A \cap B)}{P(B)},\ P(B) \neq 0.$

Rough Work